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Entropy Pooling and Bayesian Networks Applied to the Measurement of Stressed Scenarios in Credit Risk with Socioeconomic Implications

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Abstract

The socioeconomic implications of this research are profound, in that if financial institutions had robust risk measurement tools along the lines proposed herein, the massive global wealth loss and concomitant social ills (e.g., the rise of populist political ideologies) may have been far limited in severity. A critical question for banking supervisors is the correct amount of capital or liquidity resources required by an institution to support the risks taken in the course of business. The financial crises of the last several years have revealed traditional approaches such as regulatory capital ratios to be inadequate, giving rise to supervisory stress testing and scenario analysis ("ST/SA") as primary tools. A critical input into this process are macroeconomic variables that are provided by the prudential supervisors to institutions for exercises such as the Federal Reserve's Comprehensive Capital Analysis and Review ("CCAR") program. We propose an application to this exercise of a unified methodology that incorporates non-linear scenarios from alternative perspectives in a general non-normal setting for economic and market factors, which is novel in this application to credit risk and a supervisory application of ST/SA. This includes a flexible framework for causal and predictive market scenarios that combines Bayesian networks ("BNs") and entropy pooling ("EP"), with BN generating a finite set of joint causal views for the relevant risk factors, *while EP is used to project each of these stress scenarios over stochastic simulations. The joint view probabilities from BNs are naturally used as weights for the associated EP probability vectors to compute a single posterior probability distribution. The framework allows us to implement economic scenarios and perform ST/SA conditional on realizations of relevant risk factors in a purely causal and predictive manner. This procedure provides a tool for portfolio, risk and supervisory practitioners to manage credit risk and profitability. We test these methodologies empirically with aggregate banking charge-off data for several lending segments and benchmark the results against a challenger vector-autoregressive model, finding that the BN and EP approaches produce scenarios that have superior characteristics. Namely, the proposed approach produces more conservative and more accurately estimated portfolio risk measures for the same severely adverse scenario, with a more parsimonious model that has better fit to the data.*

Keywords: Stress Testing, Scenario Analysis, Credit Risk, CCAR, DFAST, Financial Crisis, Entropy Poling, Bayesian Networks, Model Risk

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Introduction

In the aftermath of the financial crisis [1,2]. Regulators have utilized stress testing to evaluate the soundness of financial institutions' risk management procedures. The primary means of risk management, particularly in the field of credit, is through advanced mathematical, statistical and quantitative techniques and models, which leads to *model risk* [3,4]. Model risk can be defined as the potential that a model does not sufficiently capture the risks it is used to assess, and the danger that it may underestimate potential risks in the future[5]. *Stress testing* and *scenario analysis* ("ST/SA") has been used by supervisors to assess the reliability of credit risk models, as can be seen in the revised Basel framework, and the *Federal Reserve's Comprehensive Capital Analysis and Review* ("CCAR") program [12,13] ST/SA allows the practitioner to explore the implications on a given portfolio of a set of subjective views on possible market realizations [14].

Prior to the financial crisis, most of the prominent financial institutions that failed (e.g., Lehman, Bear Stearns, Washington Mutual, Freddie Mac and Fannie Mae) were considered well-capitalized according to the prudential standards across a wide span of regulators. Another commonality among the large, failed firms included a general exposure to residential real estate, either directly or through securitization. Further, it is widely believed that the internal risk models of these institutions were not wildly out of line with those of the regulators [15]. We learned through these unanticipated failures that the answer to the question of how much capital an institution needs to avoid failure was not satisfactory. While capital models accept a non-zero probability of default according to the risk aversion of the institution or the supervisor, the utter failure of these constructs to even come close to projecting the perils that these institutions faced was a great motivator for considering alternative tools to assess capital adequacy, such as the ST/SA discipline.

Bank holding companies ("BHCs") face several considerations in modeling losses for wholesale and retail lending portfolios. CCAR participants face challenges in estimating losses based on scenarios and their associated risk drivers. The selection of modeling methodology must satisfy several criteria, such as suitability for portfolio type, materiality, data availability as well as alignment with chosen risk drivers. There are two broad categories of model types in use. *Bottom-up models* are loan- or obligor-level models used by banks to forecast the expected losses of retail and wholesale exposures for each loan. The expected loss is calculated for each loan, and then the sum of expected losses across all loans provides an estimate of portfolio losses, through conditioning on macroeconomic or financial / obligor specific variables. The primary advantages of bottom-up models are the ease of modeling heterogeneity of underlying loans and interaction of loan-level risk factors. The primary disadvantages of loan-level models are that while there are a variety of loan-level methodologies that can be used, these models are much more complex to specify and estimate. These models generally require more sophisticated econometric and simulation techniques, and model validation standards may be more stringent. In contrast, *top-down models* are pool (or segment) level models used by banks to forecast charge-off rates by retail and wholesale loan types as a function of macroeconomic and financial variables. In most cases for these models, banks use only one to four macroeconomic and financial risk drivers as explanatory variables. These variables are usually determined by the interaction between model development teams and line of business experts. The primary advantage of top-down models has been the ready availability of data and the simplicity of model estimation. The primary disadvantage of pool-level models is that borrower specific characteristics are generally not used as variables, except at the aggregate level using pool averages. Modeling challenges include determination of appropriate loss horizon (e.g., for CCAR it is a 9-quarter duration), determination of an appropriate averaging methodology, appropriate data segmentation and loss aggregation, as well as the annualization of loss rates.

In this study we present the EP and BN approaches to ST/SA in a novel application to credit risk and with implications for the supervisory CCAR exercise. The inputs are an arbitrary economic model referred to as a "prior", and fully general views or stress test scenarios on that economy. The output is a distribution referred to as a "posterior", that incorporates all the inputs and can be used for risk management and portfolio optimization. We obtain the posterior by interpreting the scenarios as statements that distort the prior distribution, in such a way that the least possible amount of spurious structure is imposed, where the natural index for the structure of a distribution is its entropy. Therefore, we define the posterior distribution as the one that minimizes the entropy relative to the prior, and then by opinion pooling we assign different confidence levels to different subjective scenarios. EP is amongst the techniques that accommodates non-normal economies with non-linear combinations of risk factors that impact the returns or credit losses directly or only statistically through correlations. Scenarios may refer to measures of central tendency (expectations or medians) or higher moments (volatilities, correlations and tail risk measures). Furthermore, this approach accommodates input from multiple users and multiple confidence levels for different views, as well as admitting fat tailed distributions as seen in credit return or loss. In its most general implementation as implemented herein, the reference model is represented by Monte Carlo simulations, and the posterior which incorporates all the inputs is represented by the same simulations with new probabilities, a rather flexible framework for causal and predictive ST/SA by combining BNs and EP.

Before proceeding with the remainder of this study, let us highlight the socioeconomic implications of this research. While technologies developed by financial systems can contribute to economic development by providing institutions with useful tools for risk management, when they fail to manage such risks through negligence or faulty methodologies, they can create severe financial crises with devastating social and economic effects. A case in point is the financial crisis of 2008-2009 that transformed the lives of many individuals and families (even in advanced countries) for the worse by condemning countless people to poverty and exclusion. The adverse social reverberations of this event have been reflected in phenomena such as the rise of populist political ideologies, erosion of trust in societal institutions and fragmentation of the post-World War II geopolitical order. While the great financial crisis may not have been a proximate cause of these events, it is hard to argue that it was not a significant catalyst. It is our contention that if financial institutions had techniques in their toolkit such as EP and BN, even if applied to the prevalent pre-financial crisis risk measurement regime, that the severity of the financial crisis and ensuing fallout would have been greatly ameliorated.

This paper shall proceed as follows. The following section reviews the available literature on ST/SA in credit risk as well as the EP and BN constructs. The subsequent three sections present the formalism of the EP, BN and challenger times series regression model methodologies, respectively. The following two sections presents the data used in the empirical analysis and the corresponding empirical results. The final section concludes the study and provides directions for future avenues of research.

Review of the Literature

Since the dawn of modern risk management in the 1990s, ST/SA has been a tool used to address the basic question of how exposures or positions behave under adverse conditions. Traditionally this form of ST/SA has been in the domain of *sensitivity analysis* (e.g., shocks to spreads, prices, volatilities, etc.) or *historical scenario analysis* (e.g., historical episodes such as Black Monday

1987 or the post-Lehman bankruptcy period; or hypothetical situations such as modern version of the Great Depression or stagflation). These analyses are particularly suited to market risk, where data are plentiful, but for other risk types in data-scarce environments (e.g., operational, credit, reputational or business risk) there is a greater reliance on *hypothetical scenario analysis* (e.g., natural disasters, computer fraud, litigation events, etc.).

Regulators first introduced ST/SA within the Basel I Accords, with the 1996 Market Risk Amendment [5]. Around the same time, the publication of RiskMetrics^{TM} in 1996 [16]. marked risk management as a separate technical discipline, and therein all of the above-mentioned types of ST/SA are referenced. The seminal paper on *Value-at-Risk* ("VaR"), also had a part devoted to the topic of ST/SA, while other authors provided detailed discussions of VaR-based stress tests as found largely in the trading and treasury functions [17-19]. The *Committee on Global Financial Systems* ("CGFS") conducted a survey on stress testing in 2000 that had similar findings. Another study highlighted that most of the ST/SA exercises performed to date were shocks to market observables based upon historical events, which have the advantage of being well-defined and easy to understand, especially when dealing with the trading book constituted of marketable asset classes [20].

However, in the case of the banking book (e.g., corporate / C&I or consumer loans), this approach of asset class shocks does not carry over as well, as to the extent these are less marketable there are more idiosyncrasies to account for. Therefore, ST/SA with respect to credit risk has evolved later and as a separate discipline in the domain of credit portfolio modeling. However, even in the seminal examples of CreditMetrics^{TM}, and CreditRisk+^{TM} ST/SA was not a component of such models [21,22]. The commonality of all such credit portfolio models was subsequently demonstrated, as well as the correspondence between the state of the economy and the credit loss distribution, and therefore that this framework is naturally amenable to ST/SA [23]. In this spirit, a class of models was built upon the CreditMetrics^{TM} framework through macroeconomic stress testing on credit portfolios using credit migration matrices [22]. ST/SA supervisory requirements with respect to the banking book were rather undeveloped prior to the crisis, although it was rather prescriptive in other domains, examples including the joint policy statement on interest rate risk, guidance on counterparty credit risk, as well as country risk management [24-26].

Following the financial crisis of the last decade, we find an expansion in the literature on ST/SA, starting with a survey of the then extant literature on stress testing for credit risk [27]. As part of a field of literature addressing various modeling approaches to ST/SA, we find various papers addressing alternative issues in this domain, including the aggregation of risk types and model validation [28,29,30]. Various papers have laid out the reasons why ST/SA has become such a dominant tool for regulators, including rationales for its utility, outlines for its execution, as well as guidelines and opinions on disseminating the output under various conditions [15]. This includes a survey of practices and supervisory expectations for stress tests in a credit risk framework, and presentation of simple examples of a ratings migration-based approach, using the CreditMetrics™ approach [22]. Another set of papers argues for a Bayesian approach to stress testing, having the capability to cohesively incorporate expert knowledge model design, proposing a methodology for coherently incorporating expert opinion into the stress test modeling process. In a later book, the author proposes a Bayesian casual network model, for ST/SA of a bank [31]. Finally, yet another recent study features the application of a Bayesian regression model for credit loss implemented using Fed Y9 data, wherein regulated financial institutions report their stress test losses in conjunction with Federal Reserve scenarios, which can formally incorporate exogenous factors such as such supervisory scenarios, and also quantify the uncertainty in model output that results from stochastic model inputs [32]. Another paper in this stream of literature presents an analysis of the impact of asset price bubbles on standard credit risk measures and provides evidence that asset price bubbles are a phenomenon that must be taken into consideration in the proper determination of economic capital for both credit risk management and measurement purposes [33]. The author also calibrates the model to historical equity prices and in a ST exercise projects credit losses on both baseline and stressed conditions for bubble and non-bubble parameter estimate settings.

The seminal approach in the stream of literature on ST/SA addressed herein is [34], where they add uncertainty on the views and on the reference risk model, with further generalizations having been proposed in recent years. [35] extend this model by providing an ST/SA framework for volatilities and correlations in addition to expectations. [36] incorporate optimal portfolios from ordering information into this framework. [37] explores non-normal markets, but ST/SA of correlations and non-linear scenarios are not allowed, and this framework relies on ad-hoc manipulations.

[38] processes partial views on expectations and covariances based on the principle of least discrimination. [39] extends the above models to act on risk factors instead of returns and thus covers highly non-linear derivative markets and views on external factors that influence portfolio returns only statistically. [40] first introduces the EP approach to ST/SA.

The idea of using BNs for causal market ST/SA was initially introduced by [41]. [42] introduced the idea of combing BNs with EP, discretizing the market drivers into bins and applying EP to multivariate histograms. [43] extends this framework by letting the risk factors be represented by joint scenarios using the market representation of return and risk factor scenarios having an associated probability vector, where BNs can be seen as causal joint view generators that additionally produce meaningful weights for each view's EP posterior probability vector.

Finally, for literature review, we discuss some background in the macroeconometric forecasting literature and describe our benchmark econometric model. In macroeconomic forecasting, there are four basic tasks that we set out to do: characterize macroeconomic time series, conduct forecasts of macroeconomic or related data, make inferences about the structure of the economy, and finally advise policy makers [44]. In the ST/SA application, we are mainly concerned with the forecasting and policy advisory functions, as stressed loss projections help banking risk managers and banking supervisors make decisions about the potential viability of their institutions during periods of extreme economic turmoil. Going back a few decades, these functions were accomplished by a variety of means, ranging from large-scale models featuring the interactions of many variables, to simple univariate relationships motivated by stylized and parsimonious theories (e.g., Okun's Law or the Phillips Curve). However, following the economic crises of the 1970s, most established economic relationships started to break down and these methods proved to be unreliable. In the early 1980s, a new macroeconometric paradigm started to take hold, *vector autoregression* ("VAR"), a simple yet flexible way to model and forecast macroeconomic relationships [45]. In contrast to the univariate autoregressive model [46,47], a VAR model is a multi-equation linear model in which variables can be explained by their own lags, as well as lags of other variables. As in the ST/SA application to CCAR we are interested in modeling the relationship and forecasting multiple macroeconomic variables, the VAR methodology is rather suitable to this end.

The Entropy Pooling Approach

We consider a portfolio of credit exposures where the default risk of which is driven by an N-dimensional vector of risk factors $\mathbf{X}$. Denoting by t the current time, by I_t the information currently available, and by τ the time to the investment horizon, there exists a deterministic function P that maps the realizations of $\mathbf{X}$ and the information I_t into the price $P_{t+\tau}$ of each exposure in at the horizon, expressed as

$$P_{t+\tau} = P_{\tau}\left(\mathbf{X}, I_t\right). \qquad (1)$$

This framework is completely general as a portfolio of exposures $\mathbf{X}$ can represent the changes in loan pricing factors (e.g., interest rates, credit or prepayment risk parameters), levels or volatilities of market or macroeconomic factors; and these may directly enter the pricing function or influence pricing through historical correlation. Furthermore, (1) may be expressed in closed-form, approximated by a second-order Taylor expansion or may represent a set of risk factors behind a computationally expensive full Monte-Carlo pricing function, such as interest rate values at different monitoring times for mortgage loans or their derivatives.

We assume the existence of a reference risk model, the joint distribution of the risk factors, as represented by its probability density function

$$\mathbf{X} \sim f(\mathbf{X}), \qquad (2)$$

referred to as the *prior factor distribution* in the BL model, which may be estimated from historical observations, else if the loan is traded calibrated to market prices [39]. In general, this is a model that risk managers use to perform various risk analyses, such as computation of risk measures (e.g., volatility, VaR or *conditional VaR* ("C-VaR"); the latter is also known as *expected shortfall*) of a portfolio, along with the contributions to such measures from the different sources of risk. On the other hand, portfolio managers and traders use this model to optimize their positions, specifying a subjective index of satisfaction S, such as the mean-volatility or mean-VaR (C-VaR) trade-off, or the certainty equivalent stemming from a utility function or a spectral measure [37]. Satisfaction depends both on the market distribution $f(\mathbf{X})$ through the prices $P_\tau(\boldsymbol{X}, I_t)$ and on the positions in the portfolio represented by a vector $\mathbf{w}$, where the optimal portfolio $\mathbf{w}^{\cdot}$ is defined as

$$\mathbf{w}^{\cdot} \equiv \underset{\mathbf{w} \in C}{\operatorname{argmax}} \{S(\mathbf{w}, f(\mathbf{X}))\}, \tag{3}$$

where C is a given set of investment constraints.

In the most general case, the risk manager expresses views (or "ST/SAs" in our terminology) on generic functions of the risk factors, a K-dimensional random variable $\left[g_1(\mathbf{X}), \ldots, g_K(\mathbf{X})\right]$, which unlike in the BL model need not be linear, whose joint distribution is implied by the reference model (2):

$$\mathbf{V} \equiv \mathbf{g}(\mathbf{X}) \sim \mathrm{f}(\mathbf{V}). \tag{4}$$

where these ST/SAs are statements on the risk factors that are at variance with the reference model (2), or in this probabilistic setting statements on their joint distribution. Therefore, the most detailed possible view specification is a complete, subjective joint distribution for those variables:

$$\mathbf{X} \sim \tilde{f}(\mathbf{X}) \neq f(\mathbf{X}). \tag{5}$$

However, general ST/SAs are statements on only select features of the distribution of $\mathbf{V}$. In the "classical" setting, such as the BL model, these constitute ST/SAs of the expectations $\tilde{\mathbb{E}}(\mathrm{V}_k)$ according to the new distribution $\tilde{f}(\mathbf{X})$. Since for distributions such as stable the expectation is not defined, in EP we consider ST/SAs on a more general location measure $\tilde{m}(\mathrm{V}_k)$, such as a median that may be specified as

$$\tilde{m}(\mathrm{V}_{\mathrm{k}}) \begin{matrix} > \\ = \\ < \end{matrix} m_k, \qquad k = 1,\ldots,K. \tag{6}$$

where the values m_k can be determined exogenously. If only qualitative ST/SAs are available, this may be set as in [40]:

$$\mathrm{m}_k \equiv \mathrm{m}_k\left\{\mathrm{V}_{\mathrm{k}}\right\} + \kappa\sigma\left\{\mathrm{V}_{\mathrm{k}}\right\}, \tag{7}$$

where σ is a measure of volatility in the reference model, such as the standard deviation (or in fat-tailed economies with infinite variance, the interquartile range), and κ is an ad-hoc multiplier (e.g., −2, −1, 1, and 2 for "very pessimistic", "pessimistic ", "optimistic" and "very optimistic”, respectively.) Note that the generalized BL ST/SAs (6) are not necessarily expressed as equality constraint as EP accommodates inequalities, such as ordering information:

$$\mathrm{m}_1\left\{\mathrm{V}_1\right\} \geq \mathrm{m}_2\left\{\mathrm{V}_2\right\} \geq \cdots \geq \mathrm{m}_k\left\{\mathrm{V}_k\right\}. \tag{8}$$

ST/SAs can be expressed on the volatilities as follows:

$$\tilde{\sigma}\{\mathrm{V}_{\mathrm{k}}\} \begin{matrix} > \\ = \\ < \end{matrix} \kappa\sigma\left\{\mathrm{V}_{\mathrm{k}}\right\}, \qquad k = 1,\ldots,K. \tag{9}$$

ST/SA of correlations may be expressed through convenient specifications for the correlation matrix $\tilde{\mathbb{C}}\{\mathbf{V}\}$ as homogeneous shrinkage

$$\tilde{\mathbb{C}}\{\mathbf{V}\} \equiv \rho_1\mathbf{I} + \rho_2\mathbb{C}\{\mathbf{V}\} + \rho_3\mathbf{1}\mathbf{1}', \tag{10}$$

where $0 \leq \rho_1, \rho_2\rho_3 < 1, 0 \leq \rho_1 + \rho_2 + \rho_3 \equiv 1, \mathbf{I}$ is the identity matrix and $\mathbf{1}$ is a vector of ones. We may also specify ST/SA on the tail behavior, as represented by $\tilde{Q}_{\mathbf{V}}(u)$, the quantile of V_k

according to the new distribution $\widetilde{f}_{\mathbf{V}}$, where the tail level $\widetilde{f}_{\mathbf{V}}$ is close to zero or one for the lower or upper tails, respectively. A convenient specification for tail ST/SA is

$$\widetilde{Q}_{\mathbf{V}}(u) \underset{<}{\overset{>}{=}} Q_{\mathbf{V}}(u), \tag{11}$$

where $\widetilde{Q}_{\mathbf{V}}(u)$ is the reference quantile induced by $f_{\mathbf{V}}$, or alternatively benchmark quantiles such as the normal or the Student's t. Finally for the specification of ST/SA views, lower or upper tail codependence may be expressed as $\widetilde{C}_{\mathbf{V}}(u)$, the *cumulative distribution function* ("CDF") of the copula with respect to $\mathbf{V}$ at joint threshold levels u close to zero or one, respectively. A convenient specification for this concept is

$$\widetilde{C}_{\mathbf{V}}(u) \underset{<}{\overset{>}{=}} \kappa C_{\mathbf{V}}(u), \tag{12}$$

where $C_{\mathbf{V}}(u)$ is the reference copula CDF induced by $f_{\mathbf{V}}$, or alternatively the benchmark copula as previously referenced.

The posterior distribution should satisfy the ST/SAs without inducing adding needless structure and as such should incorporate fidelity to the reference model (2). To this end we define the *relative entropy* between a generic distribution $\widetilde{f}_{\mathbf{X}}$ and a reference distribution $f_{\mathbf{X}}$

$$e\left(\widetilde{f}_{\mathbf{X}}, f_{\mathbf{X}}\right) \equiv \int \widetilde{f}_{\mathbf{X}}\left[\ln\left(\widetilde{f}_{\mathbf{X}}[x]\right) - \ln\left(f_{\mathbf{X}}[x]\right)\right] dx, \tag{13}$$

that while is not the only natural measure of the amount of structure in $\widetilde{f}_{\mathbf{X}}$, but quantifies how distorted $\widetilde{f}_{\mathbf{X}}$ is with respect to $f_{\mathbf{X}}$. It is evident that if the two distributions coincide, $e\left(\widetilde{f}_{\mathbf{X}}, f_{\mathbf{X}}\right) =$

0, and imposition of constraints upon $\tilde{f}_{\mathbf{X}}$ induces departure from $f_{\mathbf{X}}$ and $e\left(\tilde{f}_{\mathbf{X}}, f_{\mathbf{X}}\right) > 0$. Therefore, the posterior distribution is defined as

$$\tilde{f}_{\mathbf{X}} \equiv \underset{f \in \mathbf{V}}{\operatorname{argmin}} \left\{ e\left(f, f_{\mathbf{X}}\right) \right\}, \quad (14)$$

where $f \in \mathbf{V}$ stands for all the distributions consistent with the aforementioned ST/SAs.

The final stage of this construct is the expression of confidence in the ST/SAs, which if less than full requires that the posterior distribution of the factors must shrink towards the reference distribution, as readily achieved by *opinion-pooling* of the reference model and the full-confidence posterior as

$$\tilde{f}^{c}_{\mathbf{X}} \equiv (1-c) f_{\mathbf{X}} + c \tilde{f}_{\mathbf{X}}, \quad (15)$$

where the pooling parameter $c \in [0,1]$ represents the confidence level in the ST/SAs. In the extreme case when the confidence is total, the posterior is recovered, whereas in the absence of confidence the reference risk model is recovered. Opinion pooling becomes very useful in the context of a multiplicity in risk managers denoted by S expressing disparate ST/SAs that may be written as a set of full-confidence posterior distributions $f^{(s)}_{\mathbf{X}}$, $s = 1, \ldots, S$. It follows that the posterior distribution emerges as the confidence-weighted average of the individual full-confidence posteriors

$$\tilde{f}^{c}_{\mathbf{X}} \equiv \sum_{s=1}^{S} c_s f^{(s)}_{\mathbf{X}}. \quad (16)$$

These confidence levels can be linked naturally to the track-record of the respective risk manager, with the s^{th} confidence c_s set as an increasing function of the number ST/SAs and their concordance with economic outcomes. The definitions (15) and (16) follow from a probabilistic interpretation of the confidence wherein we specify different confidence levels for the different ST/SAs and integrates these within a multi-manager context. It follows that this construct reduces to specification of a probability measure on the power set of the ST/SAs, that unlike in the BL setting in which for EP the confidence in the ST/SAs (3.15) and those with respect to the volatility (9) are modeled separately, as being sure about future volatility and being uncertain about future economic realizations are two very different issues.

Finally, we discuss the limiting case where the risk manager has no views, or equivalently that in (14) $\mathbb{V}$ is the empty set, which implies that the confidence-weighted posterior distribution equals the reference model $f_{\mathbf{X}}$. In extremus, if the ST/SAs fully specify a joint distribution (5), then the minimization (14) is extraneous. It is evident from these considerations that to maintain consistency with the principle of *minimum discrimination information*, the full-confidence posterior follows from its conditional-marginal decomposition:

$$\tilde{f}_{\mathbf{X}}(x) \equiv \int f_{\mathbf{X}|\mathbf{v}}(x)\,\tilde{f}_{\mathbf{V}}(\mathbf{v})d\mathbf{v}. \qquad (17)$$

In particular, this is the case of purely hypothetical ST/SA analysis where full probability is attributed to a singular outcome $g(\mathbf{X}) \equiv \tilde{\mathbf{v}}$, or equivalently that the ST/SAs are represented with a Dirac delta centered on the scenario $\tilde{f}_{\mathbf{V}}(\mathbf{v}) \equiv \delta(\mathbf{v} - \tilde{\mathbf{v}})$. Upon substitution of this expression into (17) we recover $\tilde{f}_{\mathbf{X}} = f_{\mathbf{X}|\mathbf{v}}$, which means that the full-confidence posterior distribution is simply the reference distribution, conditioned on $g(\mathbf{X})$ assuming the ST/SA values $\tilde{\mathbf{v}}$, implying that EP includes the full-distribution specification and standard ST/SA analysis as special cases.

Causal and Predictive Framework - Bayesian Networks and Integrating Entropy Pooling

A BN is a *directed acyclic graph* ("DAG") that represents a set of variables and their conditional dependencies. Each node in a DAG represents a variable and each directed edge (arrow) represents a conditional dependency. If two nodes are not connected by a directed path, they are said to be conditionally independent. The conditional dependence between two variables in a BN does not necessarily represent a causal relationship, but as that is the usual assumption we will make it in this framework as well. An important realization is that causality is almost always an assumption in finance and economics applications. It is usually nontrivial to prove causality, and it is also the hardest part of estimating BNs. Once the conditional dependencies are specified, estimating the probabilities of a discrete BN is straightforward. In this framework, we suggest specifying both the conditional dependencies and probabilities for the BN, as this offers a high level of flexibility, allowing one to hypothesize about and impose future causal relationships that are not necessarily strongly present in historical data. However, a BN where both the causal structure and probabilities are estimated based on historical data will work just as well in this framework.

Specifying the probability tables of a discrete BN might seem like a daunting task at first, but it is usually easier than one expects if the task is approached in a structured way. The following four step procedure works well in practice:

- Specify the relevant variables (nodes) of the BN;
- specify the causal relationships (edges/arrows) between the variables;
- infer the size of the conditional probability tables; and,
- populate the conditional probability tables one at a time.

In all these steps, obviously it is useful to have an elegant implementation of BN technology to keep track of and manage all the information. It is convenient to have the tables auto generated and to be able to specify probability ranges or leaving probabilities unspecified with some method helping to estimate a probability based on the defined BN structure, as well as the use of maximum entropy for inferring missing values.

With a high-level understanding of BNs, the question remains how to use them for investment analysis as described in [48], as summarized herein. We want the relevant risk factors to correspond to the leaf nodes of the BN, for example ST/SA variables (e.g., real rate, inflation or risk premium), while the other nodes are variables that causally affect the distribution of these (e.g., GDP growth or unemployment rates.)

Letting $\mathbf{X} = \left(X_1,\ldots,X_N\right) \in \mathbb{N}^N$ denote the N-dimensional vector of random variables/nodes in a discrete BN, the joint probability can be computed using the well-known chain rule factorization

$$\mathbb{P}\left(X_1,\ldots,X_N\right) = \prod_{i=1}^{N} \mathbb{P}\left(X_i \middle| pa\left(X_i\right)\right), \quad (18)$$

where $\mathbb{P}(X_i | pa(X_i))$ denotes the probability of X_i conditional on its parents. Since root nodes do not have any parents, for I = 1 $\mathbb{P}\left(X_i \middle| pa\left(X_i\right)\right) = \mathbb{P}\left(X_i\right)$.

Letting $\mathcal{LN} \in \{1,2,\ldots,N\}$denote the leaf node indices, this framework focuses on the joint distribution of the leaf nodes X_i, $i \in \mathcal{LN}$, given by

$$\mathbb{P}\left(\left\{X_i \middle| i \in \mathcal{LN}\right\}\right) = \prod_{i \in \mathcal{LN}} \mathbb{P}\left(X_i \middle| pa\left(X_i\right)\right). \quad (19)$$

The total number of joint leaf node probabilities is given by

$$J = \prod_{i \in \mathcal{LN}} S_i, \quad (20)$$

where S_i is the number of states for node X_i, $i \in \mathcal{LN}$.

We conclude this section by discussing the integration of EP into the BN framework. We will denote the joint leaf node probabilities from (18) by p_j, $j = 1,2,\ldots,J$, and use them as weights for the associated EP posterior probability vectors. These joint leaf node probabilities are natural weight candidates for posterior EP probability vectors because $p_j \in [0,1]$ and $\sum_{j=1}^{J} p_j = 1$, which implies that the sum of the elements in $\tilde{q}_i = \sum_{j=1}^{J} p_j q_{i,j} \in \mathbb{R}^S$ is one for any set of valid EP posterior probability vectors. The EP posterior probability vectors $\tilde{q}_i$ are computed using EP for each of the joint leaf node events. Formally, each of the leaf node states $x_i = \{x_{i,1}, x_{i,2}, \dots, x_{i,S_i}\}$ is mapped into EP views $v_i = \{v_{i,1}, v_{i,2}, \dots, v_{i,S_i}\}$, $i \in \mathcal{LN}$, and combined using a Cartesian product over all v_i into J joint leaf node EP views.

Benchmark Vector Autoregression Model

Let $\mathbf{Y}_t = (Y_{1t}, \dots, Y_{kt})^T$ be a k-dimensional vector valued time series, the output variables of interest, in our application with the entries representing some loss measure in a particular segment, that may be influenced by a set of observable *input variables* denoted by $\mathbf{X}_t = (X_{1t}, \dots, X_{rt})^T$, an r-dimensional vector valued time series also referred as *exogenous variables*, and in our context representing a set of macroeconomic factors. This gives rise to the $VARMAX(p,q,s)$ ("vector autoregressive-moving average with exogenous variables") representation:

$$\mathbf{Y}_t \mathbf{\Phi}(B) = \mathbf{X}_t \boldsymbol{\Theta}(B) + \mathbf{E}_t \mathbf{\Theta}^*(B), \quad (21)$$

which is equivalent to

$$\mathbf{Y}_t - \sum_{j=1}^{p} \mathbf{\Phi}_j \mathbf{Y}_{t-j} = \sum_{j=0}^{s} \mathbf{\Theta}_j \mathbf{X}_{t-j} + E_t - \sum_{j=1}^{q} \mathbf{\Theta}_j^* \mathbf{E}_{t-j}, \quad (22)$$

where $\mathbf{\Phi}(B) = \mathbf{I}_r - \sum_{j=1}^{p} \mathbf{\Phi}_j B^j$, $\mathbf{\Theta}(B) = \sum_{j=0}^{s} \mathbf{\Theta}_j B^j$ and $\mathbf{\Theta}(B) = \mathbf{I}_r - \sum_{j=1}^{q} \mathbf{\Theta}_j^* B^j$ are *autoregressive lag polynomials* of respective orders p, s and q, respectively; and B is the *back-shift operator* that satisfies $B^i \mathbf{X}_t = \mathbf{X}_{t-i}$ for any process $\{\mathbf{X}_t\}$. It is common to assume that the input process $\mathbf{X}_t$ is generated independently of the noise process $\mathbf{E}_t = (\mathbf{E}_{1t}, \dots, \mathbf{E}_{kt})^T$. In fact, the exogenous variables $\{\mathbf{X}_t\}$ can represent both stochastic and non-stochastic (deterministic) variables, examples being sinusoidal seasonal (periodic) functions of time, used to represent the seasonal fluctuations in the output process $\{\mathbf{Y}_t\}$, or intervention analysis modeling in which a simple step (or pulse indicator) function takes the values of 0 or 1 to indicate the effect of output due to unusual intervention events in the system. The autoregressive parameter matrices Φ_j represent sensitivities of output variables to their own lags and to lags of other output variables, while the corresponding matrices Θ_j are model sensitivities of output variables to contemporaneous and lagged values of input variables. Note that the VARMAX model (21)-(22) could be written in various equivalent forms, involving a lower triangular coefficient matrix for $\mathbf{Y}_t$ at lag zero, or a leading coefficient matrix for $\mathbf{E}_t$ at lag zero, or even a more general form that contains a leading (non-singular) coefficient matrix for $\mathbf{Y}_t$ at lag zero that reflects instantaneous links amongst the output variables that are motivated by theoretical considerations (provided that the proper identifiability conditions are satisfied.) In the econometrics setting, such a model form is usually referred to as a *dynamic simultaneous equations model* or a *dynamic structural equation model*. The related model in the form of this form is obtained by multiplying the dynamic simultaneous equations model form by the inverse of the lag 0 coefficient matrix, is referred to as the *reduced form model*.

It follows that the dependency structure of the output variables $\mathbf{Y}_t$, as given by the autocovariance function, is dependent upon the parameters $\mathbf{\Phi}_j$, and hence the correlations amongst the $\mathbf{Y}_t$ as well as the correlations amongst the $\mathbf{X}_t$ that depend upon the parameters $\mathbf{\Theta}_j$. In contrast, in a system of univariate $ARMAX(p,q,s)$ ("autoregressive-moving average with exogenous variables") models, the correlations amongst the Y_t is not taken into account, hence the parameter vectors $\mathbf{\Theta}_j$ have a diagonal structure.

In this study we consider a *vector autoregressive model with exogenous variables* ("VARX"), denoted by$VARX(p,s)$, which restricts the *Moving Averag*e ("MA") terms beyond lag zero to be zero, or $\mathbf{\Theta}_j^* = \mathbf{0}_{k \times k}$ $j > 0$:

$$\mathbf{Y}_t - \sum_{j=1}^{p} \mathbf{\Phi}_j \mathbf{Y}_{t-j} = \sum_{j=1}^{s} \mathbf{\Theta}_j \mathbf{X}_{t-j} + \mathbf{E}_t \quad (23)$$

The rationale for this restriction is three-fold. First, the MA terms were in no cases significant in the model estimations, so that the data simply does not support a VARMA representation. Second, the VARX model avails us of the very convenient DSE package in R, which has computational and analytical advantages. Finally, the VARX framework is more practical and intuitive than the more elaborate VARMAX model and allows for superior communication of results to practitioners.

Description of the Data

In this section we describe some background on the supervisory CCAR exercise and the data used in this study[1]. As part of the Federal Reserve's CCAR stress testing exercise, U.S. domiciled top-tier BHCs are required to submit comprehensive capital plans, including pro forma capital analyses, based on at least one BHC defined adverse scenario. The adverse scenario is described by quarterly trajectories for key *macroeconomic variables* ("MVs") over the next nine quarters or for thirteen months to estimate loss allowances. In addition, the Federal Reserve generates its own supervisory stress scenarios, so that firms are expected to apply both BHC and supervisory stress scenarios to all exposures, to estimate potential losses under stressed operating conditions. Firms engaged in significant trading activities (e.g., Goldman Sachs or Morgan Stanley) are asked to estimate a one-time trading-related market and counterparty credit loss shock under their own BHC scenarios, and a market risk stress scenario provided by the supervisors. Large custodian banks are asked to estimate a potential default of their largest counterparty. In the case of supervisory stress scenarios, the Federal Reserve provides firms with global market shock components that are one-time, hypothetical shocks to a large set of risk factors.

In the 2025 CCAR, the Federal Reserve defined the domestic stress supervisory scenarios using the MVs[2]:

- Real GDP Growth ("RGDPG")
- Nominal GDP Growth ("NDPG")

- Consumer Price Index Inflation Rate (“CPI”)
- Real Disposable Personal Income Growth (“RDPI”)
- Nominal Disposable Personal Income Growth (“NDPI”)
- Unemployment Rate (“UNP”)
- Three-month Treasury Bill Rate (“3MTBR”)
- Five-year Treasury Bond Rate (“5YTBR”)
- Ten-year Treasury Bond Rate (“10YTBR”)
- BBB Corporate Rate (“BBBCR”)
- Mortgage Rate (“MR”)
- Prime Rate (“PR”)
- Dow Jones Index (“DJI”)
- National House Price Index (“HPI”)
- Commercial Property Price Index (“CPI”)
- Personal Consumption Expenditure Price Index (“PCE”)
- Market Volatility Index (“VIX”)

1. All data in this study are sourced the St. Louis Fed’s FRED database: https://fred.stlouisfed.org/"Federal Reserve Economic Data | FRED | St. Louis Fed.
2. https://www.federalreserve.gov/publications/2025-stress-test-scenarios.htm#xnotesregarding-scenariovariables-e87d2c09"The Fed - 2025 Stress Test Scenarios

For the purposes of this research, let us consider the supervisory scenarios in 2025, as well as the following calculated interest rate spreads:

- 10 Year Treasury minus 3 Month Treasure Spread or Term Spread (“TS”)
- BBB Corporate Rate minus 5 Year Treasury Spread or Corporate Spread (“CS”)

Therefore, we consider a diverse set of macroeconomic drivers representing varied dimensions of the economic environment and a sufficient number of drivers (balancing the consideration of avoiding over-fitting) by industry standards (i.e., at least 2-3 and no more than 5-7 independent variables). Our model selection process imposed the following criteria in selecting MVs :

- Transformations of chosen variables should indicate stationarity;
- signs of coefficient estimates are economically intuitive;
- probability values of coefficient estimates indicate statistical significance at conventional confidence levels;
- residual diagnostics indicate white noise behavior;
- model performance metrics (goodness-of-fit, risk ranking and cumulative error measures) are within industry accepted thresholds of acceptability; and,
- scenarios rank order intuitively (i.e., severely adverse scenario stress losses exceeding scenario base expected losses).

Similarly, we identify the following loss segments (with loss measured by Gross Charge-off Rates – “GCOs”) according to the same criteria, in conjunction with the requirement that they cover the most prevalent portfolio types in typical traditional banking institutions:

- Commercial and Industrial (“C&I”)
- Commercial Real Estate (“CRE”)
- Consumer Credit (“CONS”)
- Residential Mortgage (“RESI”)

Based upon the above model building principles, we identify five MVs for the purposes of this study: UNP, GDP, TS, VIX and PCE. In all cases, the transformation chosen for the MEVs is a year-on-year percentage change, with no lags. In the case of the GCOs, we examine the annualized level of the rate, as well as an annualized negative year-on-year change in the rate, where the latter is a proxy for a return on a portfolio of loans in the banking book that is used in the EP and BN analysis.

This historical data, 135 quarterly observations from 4Q91 to 3Q24 are summarized in Table 1 in terms of distributional statistics and in Table 2 in terms of correlations. We observe that the main features in the dependency structure within the group of input macroeconomic variables and the output loss rate variables, as well as the cross-correlations between these groups, all have intuitive signs and magnitudes that suggest significant relationships, and within groups the correlations are not large enough to suggest any issues with multicollinearity. In particular, we find that the levels of the quarterly charge-off rates are positively (inversely) correlated with UNP and VIX (GDP, TS and PCE), and the opposite holds true for the negative annual changes. Figures 1 through 9 show the time series plots and histograms of this data. In Table 3 we show the estimation results of the benchmark VARMAX model. We find that the optimal model has 1-lag for the endogenous proxy credit returns, only contemporaneous exogenous macroeconomic variables and no moving average terms. All coefficient estimates are of intuitive sign and statistically significant, including the estimates for the residual covariance matrix, consistent with the correlations in Table 2.

Table 1: Summary Statistics of Historical Y9 Credit Loss Rates and Federal Reserve Macroeconomic Variables (Q491—Q324)

			Mean	Standard Deviation	Minimum	1st Percentile	5th Percentile	Median	95th Percentile	99th Percentile	Maximum	Skewness	Kurtosis
Charge-offs	Levels	Commercial & Industrial	0.67%	0.58%	0.12%	0.12%	0.21%	0.43%	1.94%	2.37%	2.57%	1.4854	1.2309
		Commercial Real estate	0.41%	0.61%	-0.02%	-0.01%	0.01%	0.13%	1.80%	2.39%	2.76%	1.9698	3.2164
		Credit Card	4.46%	1.67%	1.65%	1.75%	2.98%	4.19%	7.71%	10.47%	10.51%	1.6836	3.8660
		Residential Mortgage	0.37%	0.62%	-0.04%	-0.04%	-0.01%	0.12%	1.85%	2.45%	2.78%	2.2221	3.9424
	Negative Changes	Commercial & Industrial	0.02%	0.37%	-1.40%	-1.23%	-0.37%	0.00%	0.69%	0.84%	0.92%	-0.4460	2.4853
		Commercial Real Estate	0.02%	0.28%	-1.13%	-1.09%	-0.10%	0.01%	0.44%	0.67%	0.74%	-1.4550	5.9051
		Credit Card	-0.01%	1.08%	-3.93%	-3.20%	-1.16%	0.00%	1.90%	3.18%	3.57%	-0.0752	2.8385
		Residential Mortgage	0.00%	0.28%	-1.36%	-0.93%	-0.11%	0.01%	0.38%	0.78%	1.00%	-1.2356	7.6821
Macroeconomic Variables		Unemployment Rate	-0.04%	1.55%	-7.10%	-3.32%	-1.00%	-0.30%	2.58%	4.78%	9.40%	1.6049	13.5560
		Gross Domestic Product	2.52%	2.05%	-7.50%	-3.72%	0.72%	2.66%	4.82%	5.56%	12.24%	-0.7742	7.9790
		Term Spread	1.52%	1.28%	-1.67%	-1.36%	-0.19%	1.56%	3.36%	3.61%	3.61%	-0.3391	-0.5836
		Voltility Index	-0.22%	8.09%	-35.53%	-21.92%	-8.61%	-0.28%	11.15%	19.16%	36.57%	0.0112	5.2513
		Personal Consumption Expenditure Price index	1.83%	1.08%	0.59%	0.73%	1.06%	1.51%	4.66%	5.83%	5.93%	2.5221	6.0883

Table 2: Correlations of Historical Y9 Credit Loss Rates and Federal Reserve Macroeconomic Variables (Q491—Q324)

		Unemployment Rate	Gross Domestic Product	Term Spread	Voltility Index	Personal Consumption Expenditure Price Index
Levels	Commercial & Industrial	66.75%	-52.33%	-43.94%	27.24%	-55.48%
	Commercial Real Estate	35.48%	-18.03%	-64.35%	16.37%	-17.77%
	Credit Card	42.24%	-26.44%	-52.01%	22.26%	-54.94%
	Residential Mortgage	15.10%	-18.58%	-14.59%	15.27%	-35.36%
Negative Changes	Commercial & Industrial	-42.31%	44.85%	73.35%	-31.14%	12.10%
	Commercial Real Estate	-53.94%	45.64%	70.33%	-13.26%	15.19%
	Credit Card	-17.21%	20.42%	71.05%	-11.18%	17.28%
	Residential Mortgage	-24.66%	30.07%	45.15%	-21.63%	51.53%

Figure 1: Time Series Plot and Histogram of Y9 Credit Loss Rates for Commercial and Industrial Loans (Q491—Q324)

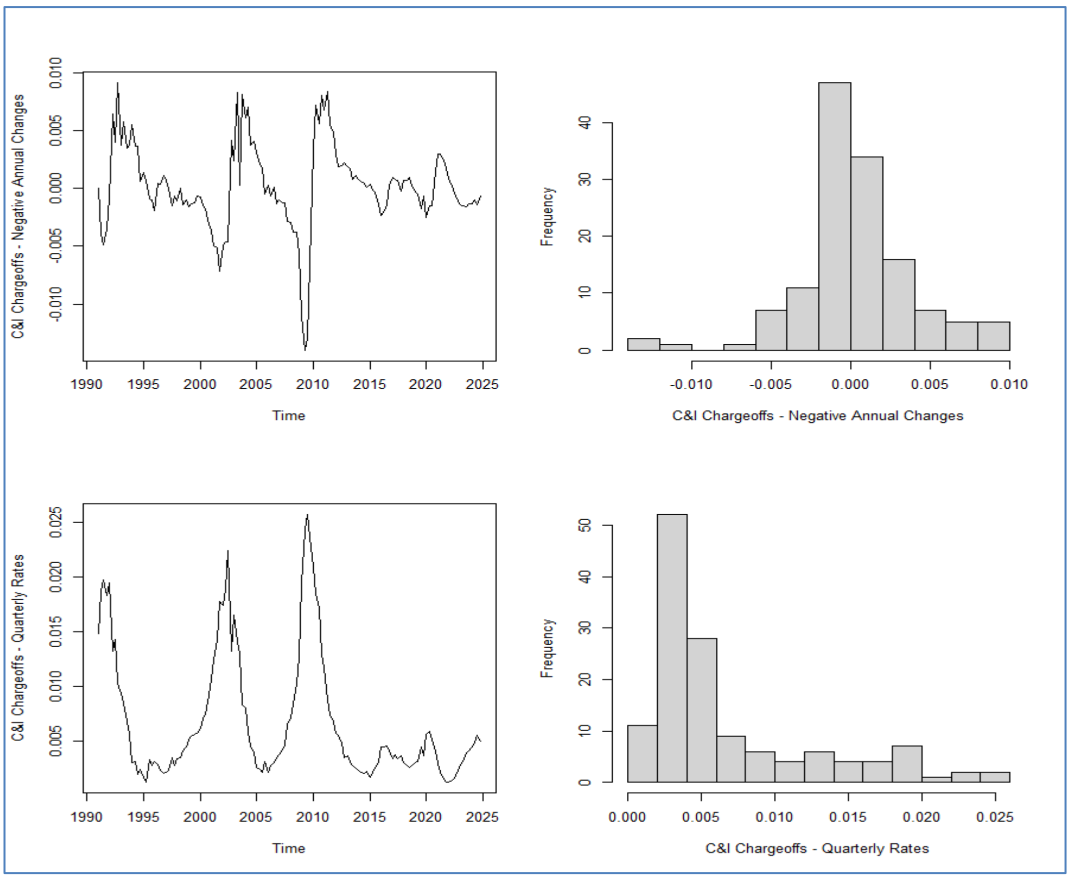


Figure 2: Time Series Plots and Histograms of Y9 Credit Loss Rate for Commercial Real Estate Loans (Q491—Q324)

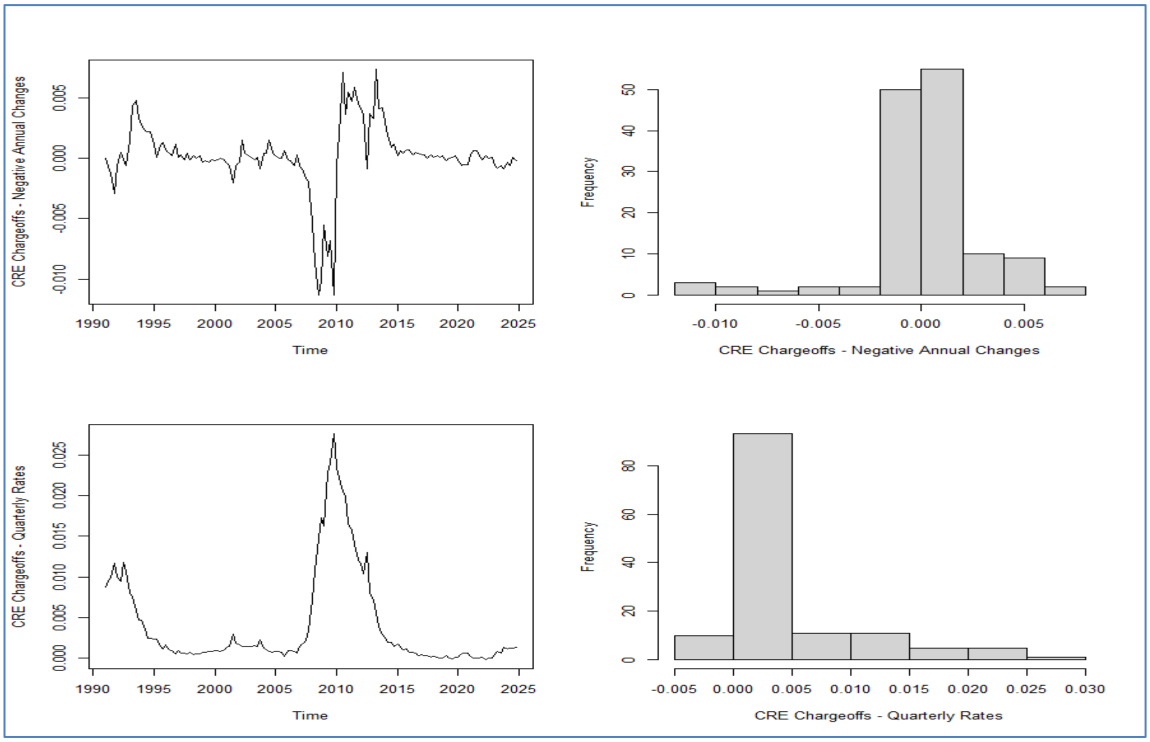


Figure 3: Time Series Plots and Histograms of Y9 Credit Loss Rate for Credit Card Loans (Q491—Q324)

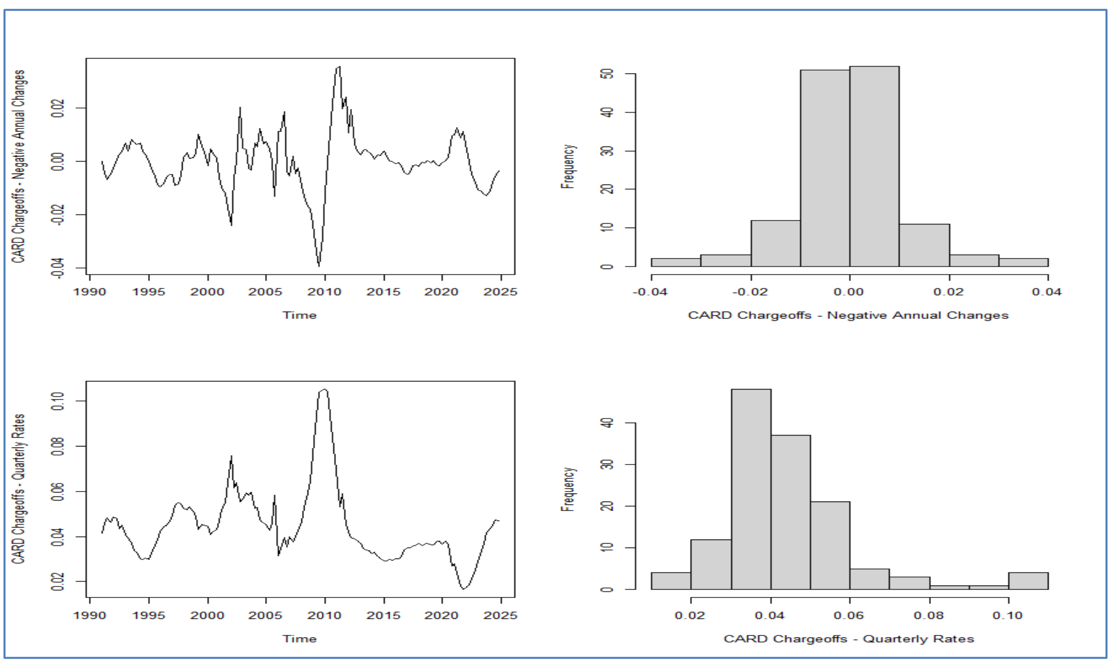


Figure 4: Time Series Plots and Histograms of Y9 Credit Loss Rate for Residential Loans (Q491—Q324)

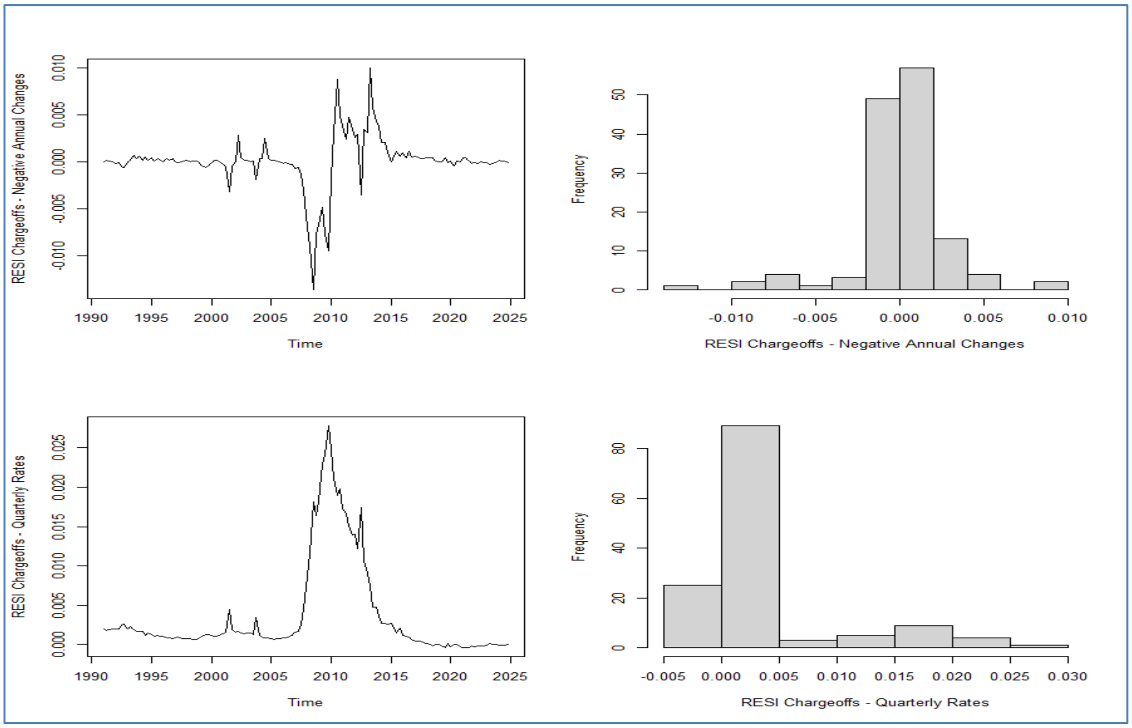


Figure 5: Time Series Plots and Histograms of Historical Federal Reserve Macroeconomic Variable – Unemployment Rate (Q491—Q324)

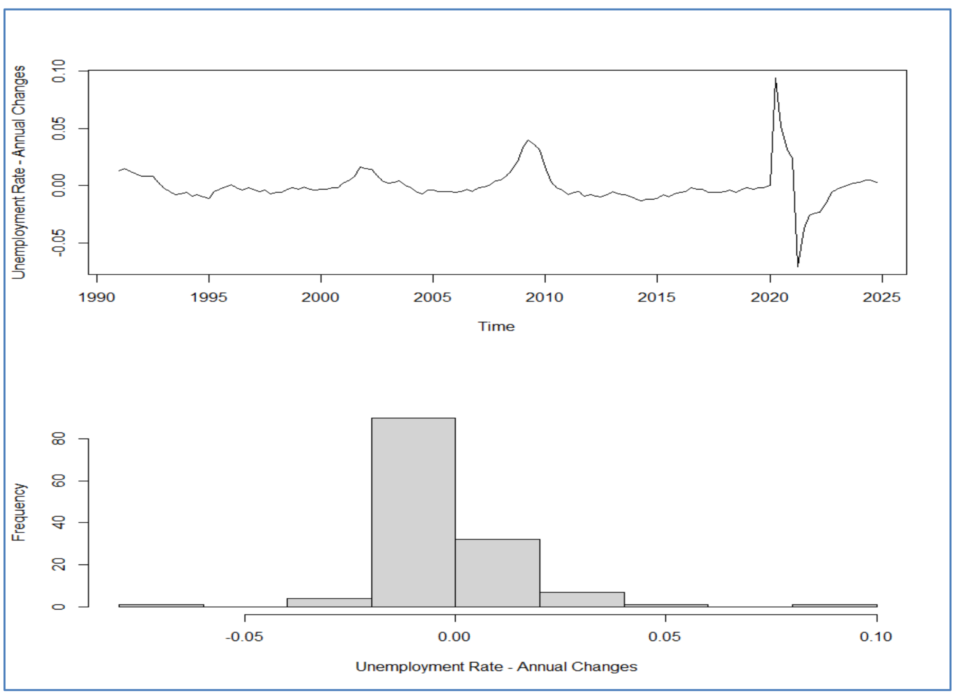


Figure 6: Time Series Plots and Histograms of Historical Federal Reserve Macroeconomic Variable – Gross Domestic Product (Q491—Q324)

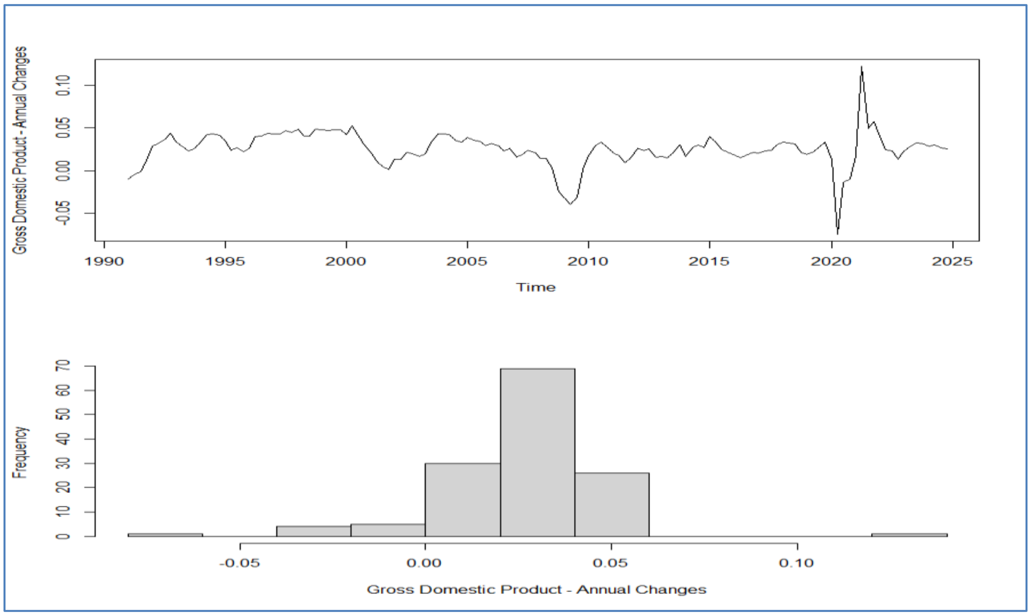


Figure 7: Time Series Plots and Histograms of Historical Federal Reserve Macroeconomic Variable – Term Spread (Q491—Q324)

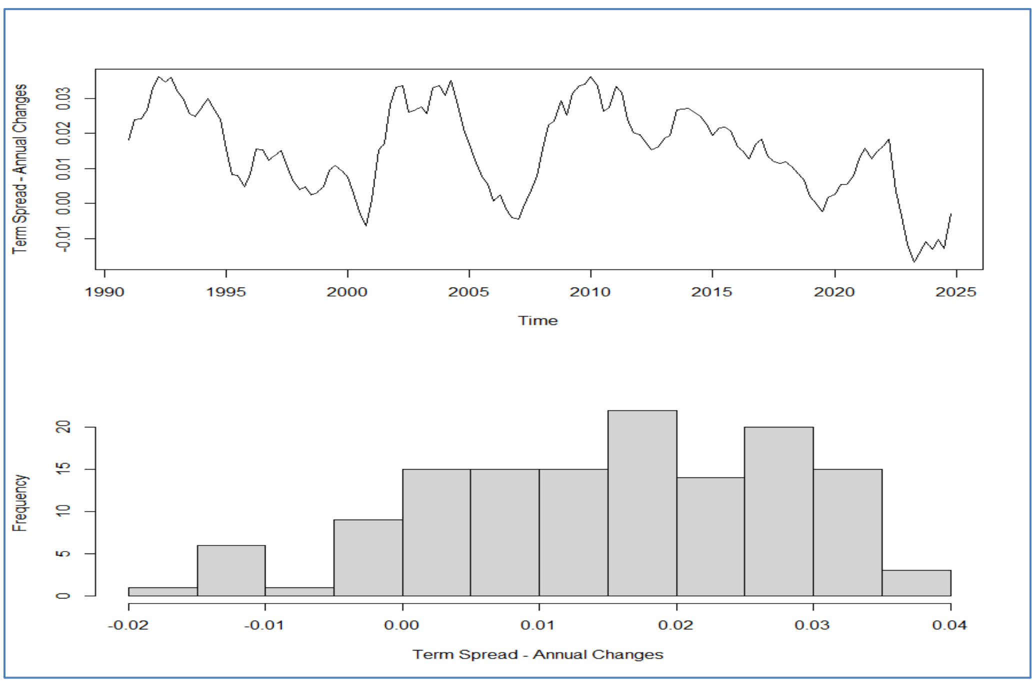


Figure 8: Time Series Plots and Histograms of Historical Federal Reserve Macroeconomic Variable – Equity Price Volatility Index (Q491—Q324)

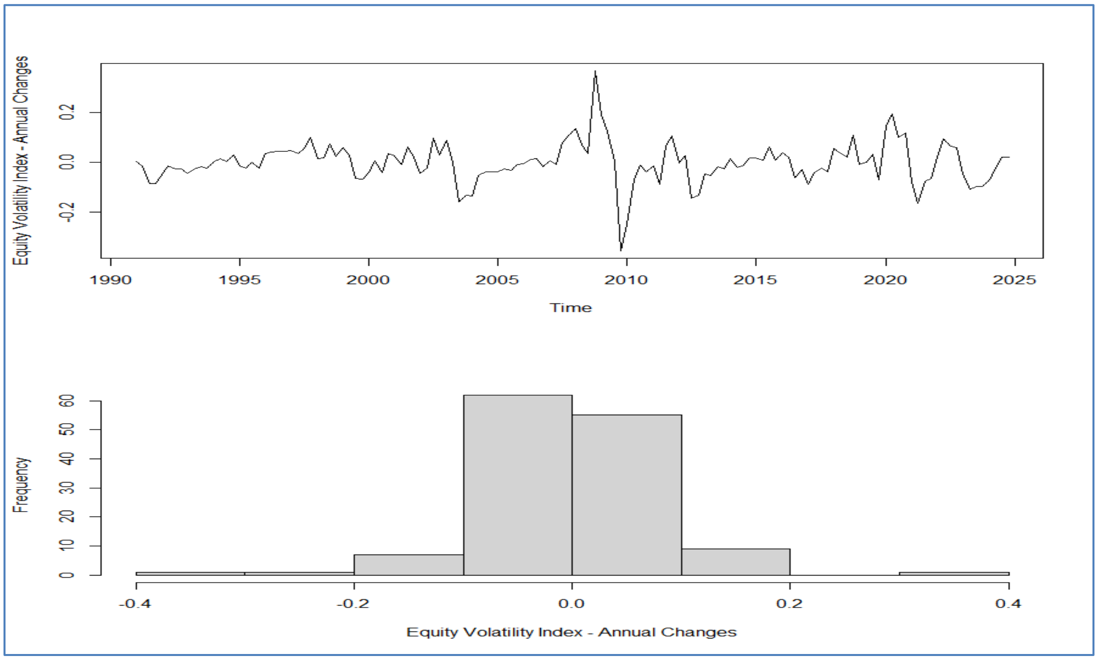


Figure 9: Time Series Plots and Histograms of Historical Federal Reserve Macroeconomic Variable – Personal Consumption Expenditure Price Index (Q491—Q324)

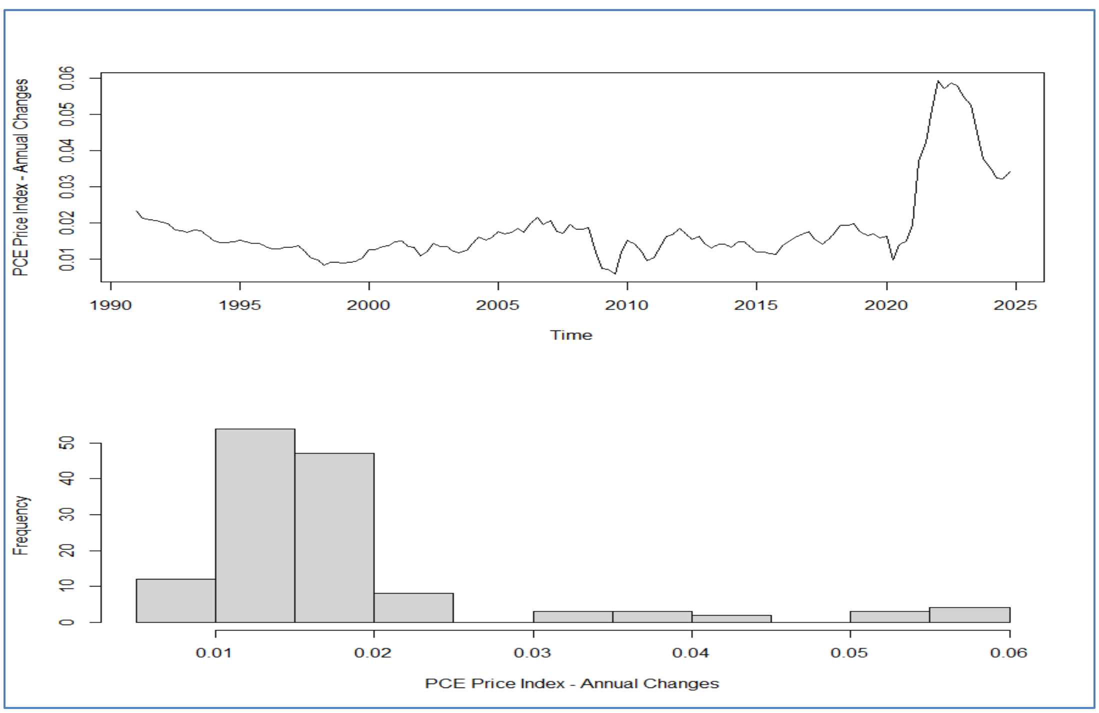


Table 3: Benchmark Vector Autoregressive Model Estimation Results - Historical Y9 Credit Loss Rates and Federal Reserve Macroeconomic Variables (Q491—Q324)

		C&I		CRE		CARD		RESI	
		Estimate	P-Value	Estimate	P-Value	Estimate	P-Value	Estimate	P-Value
	Intercept	0.0021	0.0990	0.0013	0.0334	0.0003	0.9490	0.0012	0.4140
Endogenous Variables (Lag 1)	C&I	0.5099	0.0000	0.2673	0.0400	0.5505	0.0920	0.2364	0.1890
	CRE	0.2387	0.0529	0.2996	0.0167	0.1028	0.8990	0.0303	0.9100
	CARD	0.0392	0.0469	0.0339	0.2710	0.5465	0.0000	0.0529	0.4220
	RESI	0.1341	0.0346	0.2916	0.0240	0.6464	0.3900	0.6171	0.0020
Exogenous Variables (Lag 0)	UNP	-0.0230	0.0375	-0.0463	0.0151	-0.0697	0.0154	-0.0385	0.0376
	GDP	0.0404	0.0115	0.0203	0.0220	0.0040	0.0097	0.0197	0.0186
	TS	0.0476	0.0112	0.0166	0.0195	0.0085	0.0091	0.0207	0.0148
	VIX	-0.0070	0.0104	-0.0012	0.0168	-0.0070	0.0109	-0.0008	0.0161
	PCE	0.0248	0.0290	0.0280	0.0163	0.0398	0.0104	0.0252	0.0147
Covariance Matrix	C&I	0.1900	0.0000						
	CRE	0.1000	0.0000	0.1400	0.0000				
	CARD	0.1200	0.0010	0.1000	0.0010	0.0980	0.0040		
	RESI	0.0939	0.0000	0.1300	0.0000	0.1000	0.0000	0.1800	0.0010
Model Fit Statistics	Log-Likelihood	2624.25							
	AIC	-5148.49							
	BIC	-5003.23							
	HQIC	-5089.46							

Empirical Results

In this section we present the empirical results of this study. First, we apply EP to a portfolio return analysis, in which we express views on the MV risk factors. We base our shocks on the Fed CCAR 2025 severely adverse scenarios, where for each risk factor we scale the posterior means and variances by the ratios of the means and variances historically and in the severely adverse scenario, respectively, the summary statistics and time series trajectories of which are shown in Table 4 and in Figure 10, respectively. In Figures 11 and 12 we show the forecast trajectories under the benchmark VARMAX and EP models, respectively, while in Figure 13 we show a comparison of the portfolio distributions under equal weighting. Table 5 shows the volatility, VaR and C-VaR risk measures for the prior, EP and VARMAX model return distributions.

We first discuss a comparison of the model fit between the EP and VARMAX models. In the case of the EP model, we compute an approximation to the *Bayesian Information Criterion* (“BIC”) based upon a *deviance* and effective number of parameters measure. We measure the deviance by comparing the historical return distribution of an equally weighted portfolio with the prior distribution obtained from the EP model simulation, while the effective number of parameters in the latter is taken to be the number of sample moments of the risk factors and returns (means, standard deviations and correlations) that are inputs into the simulation. In Table 3 we see that the VARMAX model has a mean BIC of -5003.23, while we calculate a mean BIC of -6367.15 for the EP model (and we note that the other characteristics of the distributions, such as the standard deviation and tail shapes, support the superiority of the latter vs. the former), which shows materially superior fit.

It can be seen in Figure 13 that under the stressed scenarios, the portfolio return distribution shifts markedly to the left, with this change accentuated for the EP as compared to the VARMAX model. In Table 5 we observe that the volatility of the portfolio is 1.80% in the prior distribution, 2.40% in the VARMAX model and 2.86% in the EP model. This rank ordering is preserved, albeit on the proportional basis the differences somewhat narrow, for the tail risk measures: VaR of 5.70% (5.90%) in the VARMAX (EP) model as compared to 4.10% in the prior distributions, and C-VaR of 5.90% (6.84%) in the VARMAX (EP) model as compared to 5.40% in the prior distributions.
Finally, we measure the accuracy and robustness of the C-VaR risk measure through a bootstrapping exercise, where we resample with replacement the historical data 100,000 times and examine the distributions of the risk measure. We observe in Figure 14 that the EP model produces consistently higher C-VaR estimates than the VARMAX model, as the distributions are materially separated, and moreover the variability of the latter is significantly greater than the former. In Figure 15 we perform a similar bootstrap exercise, recording the BIC in each resampling run, and we reach the conclusion that on this basis the EP model outperforms the VARMAX model in terms of fit to the data.

In the next exercise, with the EP model with ST/SA upon the risk factors in hand (disregarding the VARMAX model, which we have seen is clearly sub-optimal), we explore the effect of adding ST/SA on the return distributions, above and beyond the stressing of the macroeconomic factors. This is important, in that in managing capital for CCAR purposes, banks often impose overlays or qualitative factor adjustments, to account for portfolio level idiosyncratic risks not captured in stressed scenarios. Based upon communications with portfolio management experts, we impose the following ST/SA views on the asset class returns:

- Commercial and Industrial: a return of -1.30% and volatility of no less than 0.80%;
- Commercial Real Estate: a return of -1.00% and volatility of no less than 0.060%;
- Consumer Credit: a return of -3.5% and volatility of no less than 2.00%; and,
- Residential Mortgage: a return of -1.0% and volatility of no less than 0.60%.

The prior and posterior distributions of negative annual changes in Y9 charge-off rates are shown in Figures16 through 19. The results of this analysis are shown in Table 6, conditional C-VaR measures of posterior portfolio returns for equal and minimum risk weighted for risk factor shocks, as well as minimum risk weighted returns for in addition shocks to annual changes in Y9 charge-off rates. We see that for the risk factor shocked case, minimizing risk along the efficient frontier reduces C-VaR by 44bps, and increases (decreases) the weighting toward C&I and CRE by about 10% and 7% (Card and Resi by about 13% and 3%), respectively, which is intuitive as the movements are toward asset classes with lower return volatility relative to mean return. When we overlay the views on the asset class returns in the minimum risk case the C-VaR increases by 80bps to 7.18, but the portfolio weights deviate back toward the equal weighted scheme, albeit with main-

Table 4: Summary Statistics of FED CCAR 2025 Severely Adverse Macroeconomic Variables (Q125—Q427)

		Mean	Standard Deviation	Minimum	1st Percentile	5th Percentile	Median	95th Percentile	99th Percentile	Maximum	Skewness	Kurtosis
Macroeconomic Variables	Unemployment Rate	0.88%	2.01%	-1.40%	-1.39%	-1.28%	0.80%	3.72%	3.86%	3.90%	0.2181	-1.6976
	Gross Domestic Product	-21.10%	47.65%	-111.00%	-110.28%	-103.40%	3.00%	8.36%	8.55%	8.60%	-1.4471	0.1498
	Term Spread	0.35%	0.48%	-0.50%	-0.43%	0.12%	0.20%	1.22%	1.36%	1.40%	0.9068	1.6352
	Voltility Index	1.65%	21.11%	-18.60%	-18.13%	-14.10%	-5.80%	44.78%	45.60%	45.80%	1.5533	1.2376
	Personal Consumption Expenditure Price	-0.23%	0.63%	-1.80%	-1.74%	-1.18%	0.10%	0.10%	0.10%	0.10%	-1.8765	2.4852

Figure 10: Time Series Trajectories of FED CCAR 2025 Severely Adverse Macroeconomic Variables (Q125—Q427)

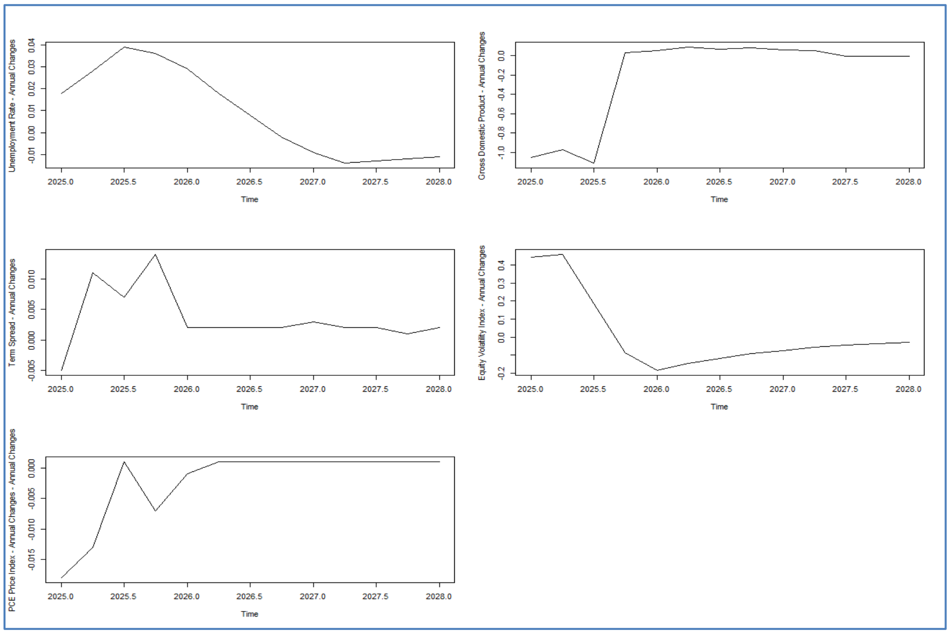


Figure 11: Time Series Trajectories of Forecasted Negative Annual Changes in Y9 Charge-off Rates in the Benchmark Vector-Autoregressive Model under FED CCAR 2025 Severely Adverse Macroeconomic Variables (Q125—Q427)

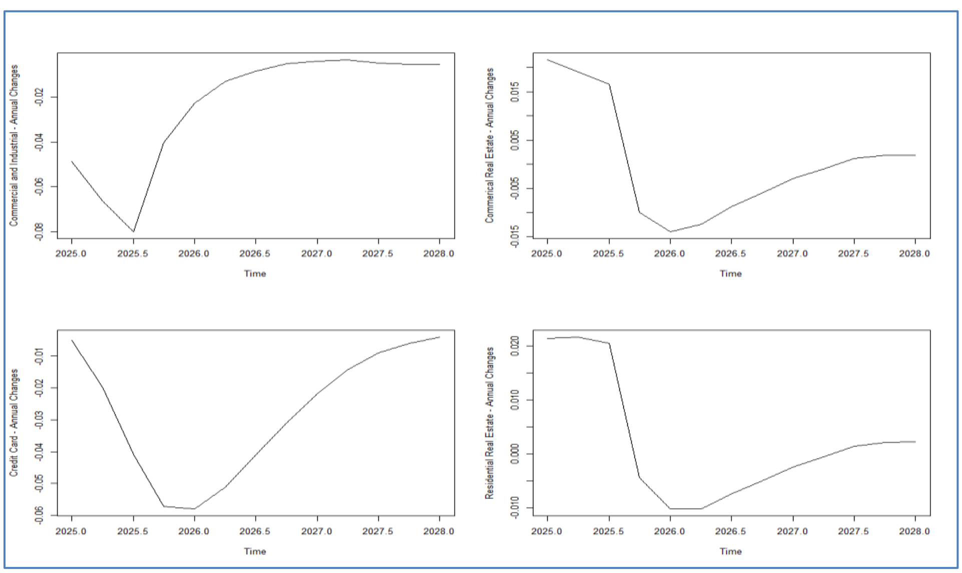


Figure 12: Time Series Trajectories of Forecasted Negative Annual Changes in Y9 Charge-off Rates in the Entropy Pooling Model under FED CCAR 2025 Severely Adverse Macroeconomic Variables (Q125—Q427)

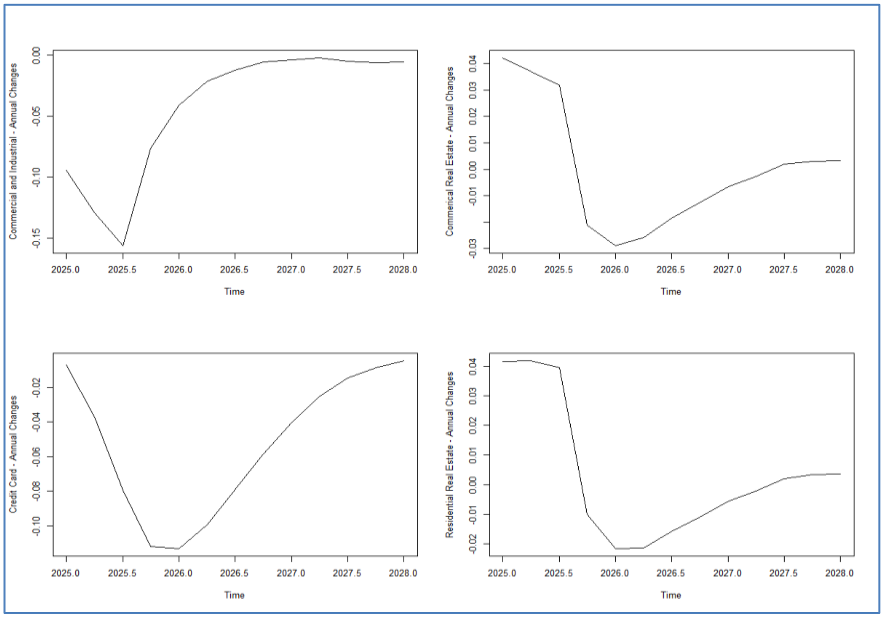


Figure 13: Equal Weighted Portfolio Loss Return Distributions of Forecasted Negative Annual Changes in Y9 Charge-off Rates in the Entropy Pooling and Benchmark VARMAX Models under FED CCAR 2025 Severely Adverse Macroeconomic Variables (Q125—Q427)

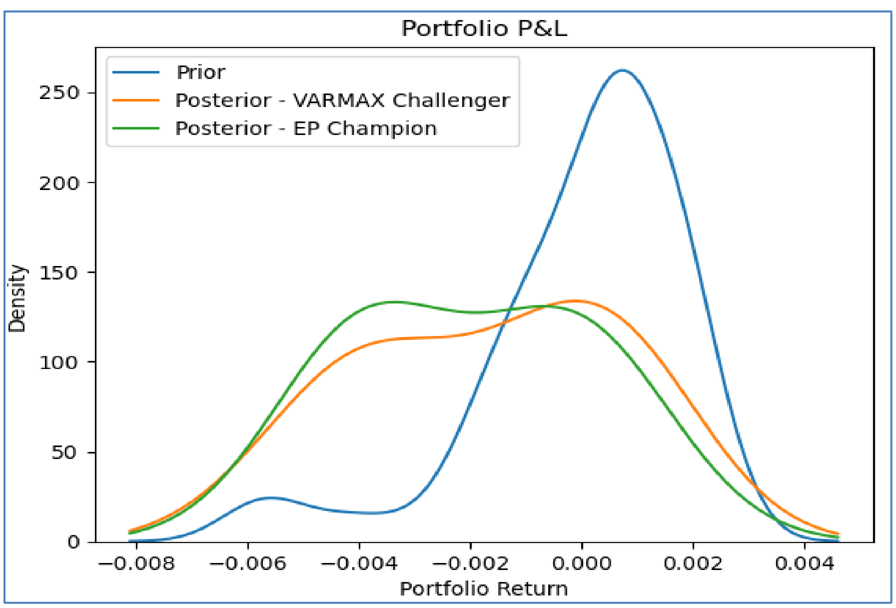


Table 5: Risk Measures of Equal Weighted Portfolio Loss Return Distributions of Forecasted Negative Annual Changes in Y9 Charge-off Rates in the Entropy Pooling and Benchmark VARMAX Models under FED CCAR 2025 Severely Adverse Macroeconomic Variables (Q125—Q427)

Volatility	Prior Distribution	1.80%
	Entropy Polling Model	2.86%
	Vector Autortegressive Model	2.40%
Value-at-Risk	Prior Distribution	4.10%
	Entropy Polling Model	5.88%
	Vector Autortegressive Model	5.70%
Conditional Value-at-Risk	Prior Distribution	5.40%
	Entropy Polling Model	6.84%
	Vector Autortegressive Model	5.90%

Figure 14: Resampled Distributions of Conditional Value-at-Risk Measures of Equal Weighted Portfolio Loss Return Distributions of Forecasted Negative Annual Changes in Y9 Charge-off Rates in the Entropy Pooling and Benchmark VARMAX Models under FED CCAR 2025 Severely Adverse Macroeconomic Variables (Q125—Q427)

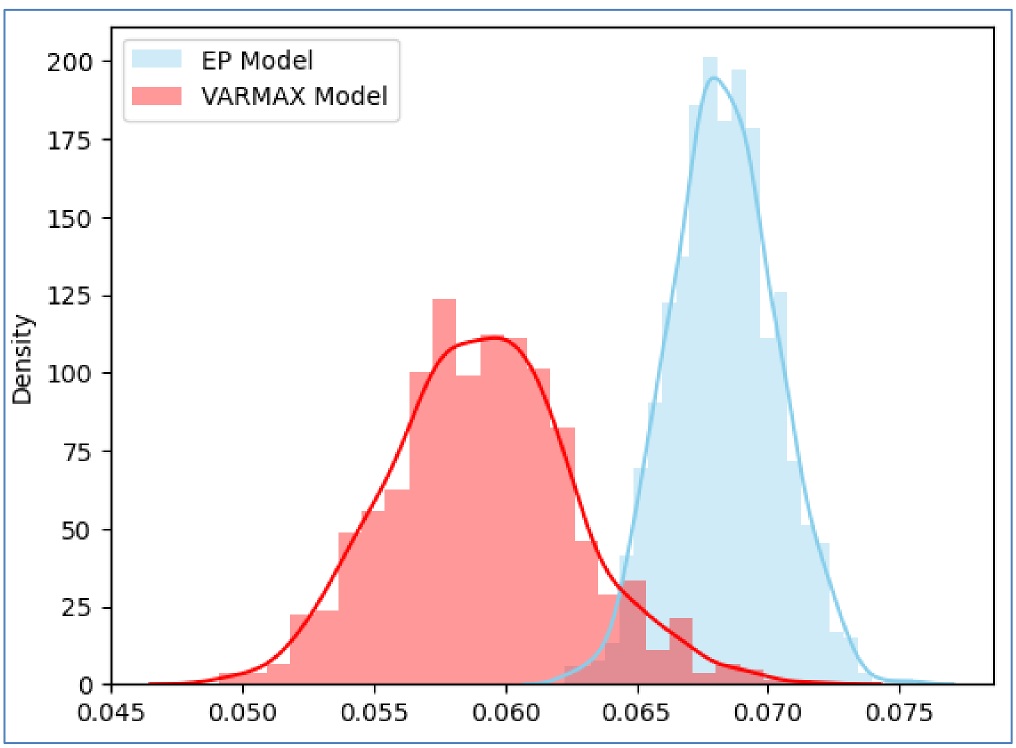


Figure 15: Resampled Distributions of Bayesian Information Criterion Model Fit Measures in the Entropy Pooling and Benchmark VARMAX Models under FED CCAR 2025 Severely Adverse Macroeconomic Variables (Q125—Q427)

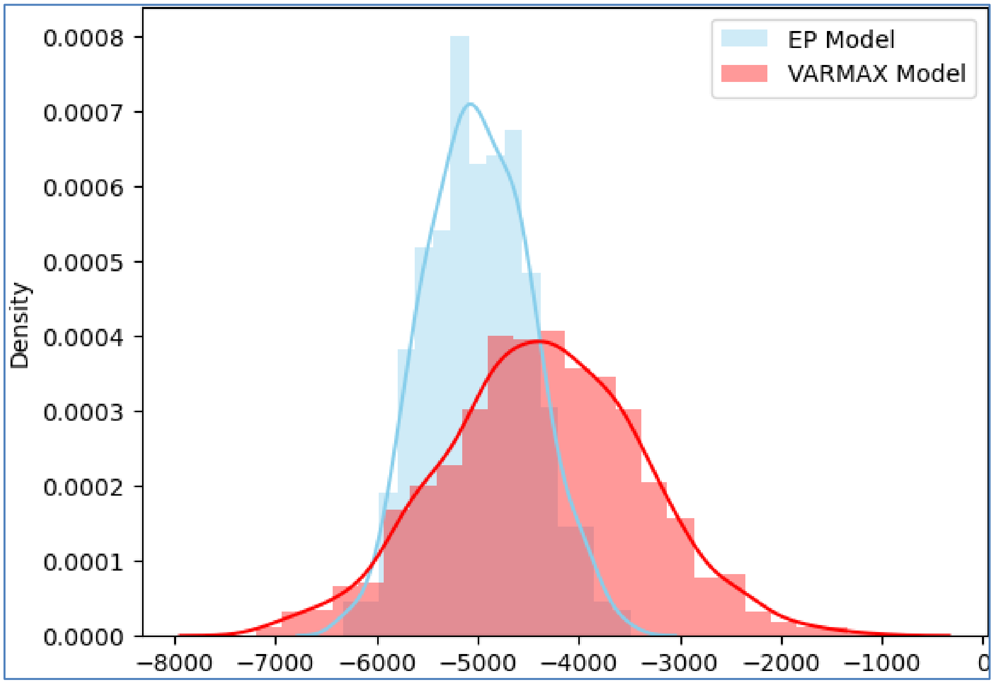


Figure 16: Entropy Pooling Model Prior and Posterior Distributions of Negative Annual Changes in Y9 Charge-off Rates for Commercial and Industrial Loans (Q491—Q324)

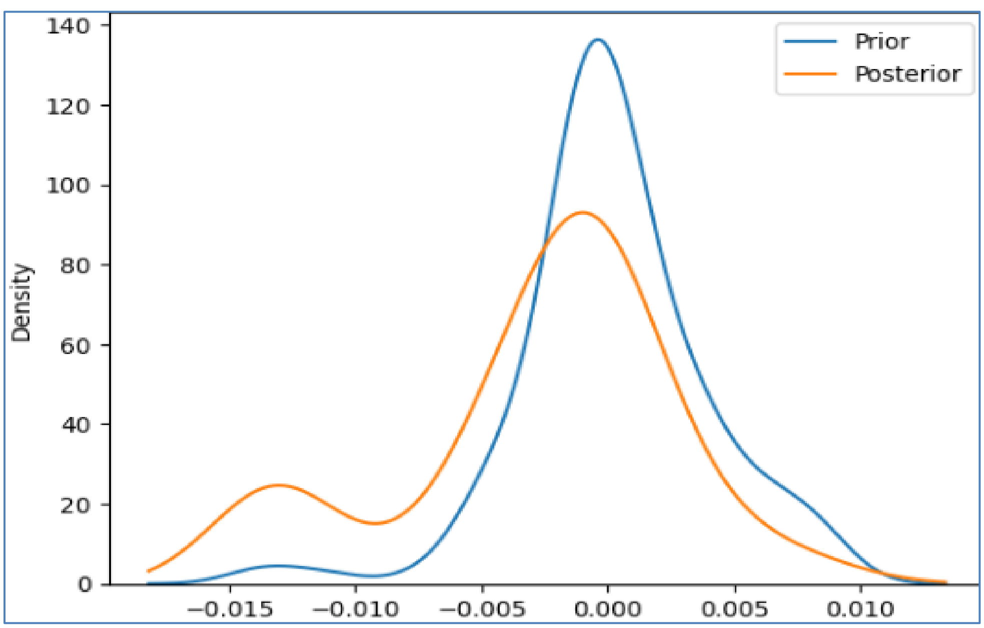


Figure 17: Entropy Pooling Model Prior and Posterior Distributions of Negative Annual Changes in Y9 Charge-off Rates for Commercial Real Estate Loans (Q491—Q324)

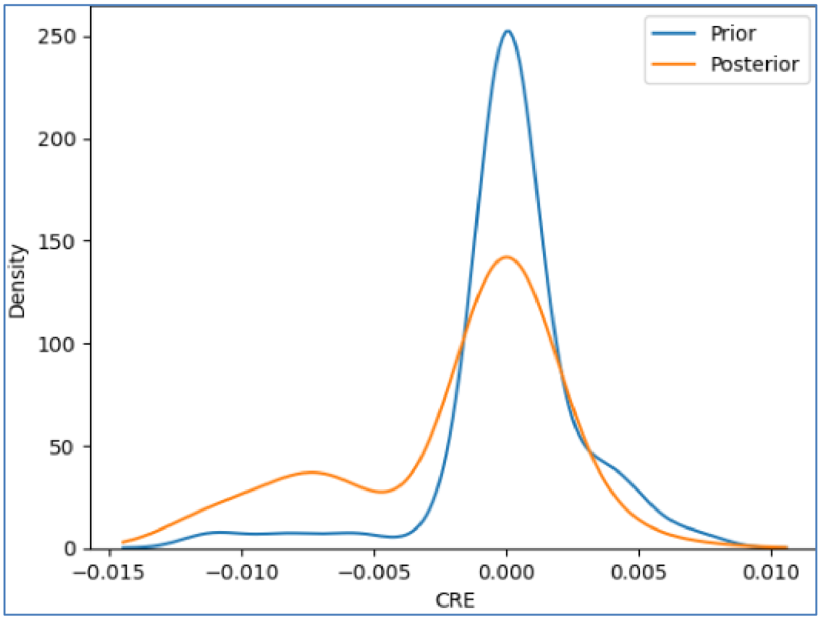


Figure 18: Entropy Pooling Model Prior and Posterior Distributions of Negative Annual Changes in Y9 Charge-off Rates for Consumer Loans (Q491—Q324)

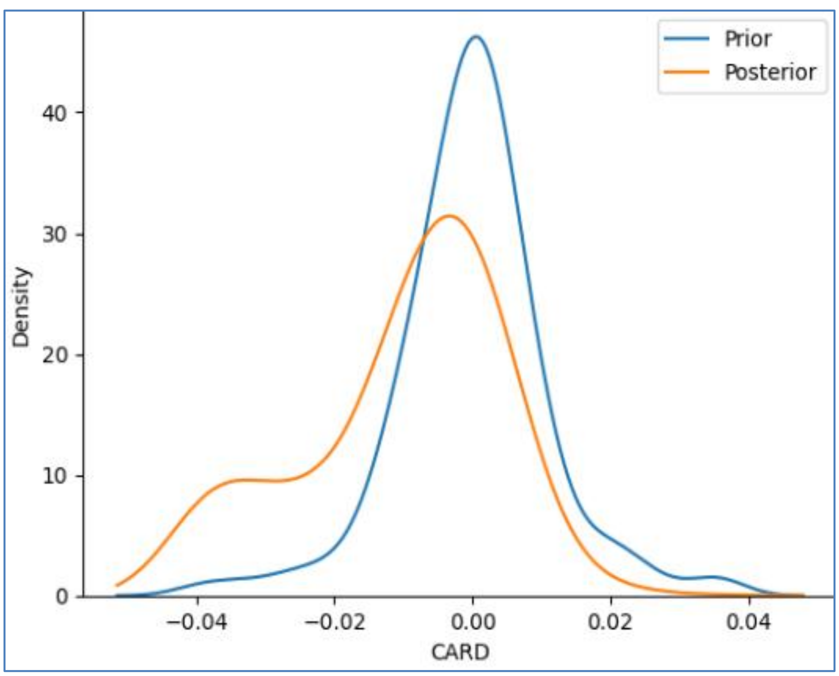


Figure 19: Entropy Pooling Model Prior and Posterior Distributions of Negative Annual Changes in Y9 Charge-off Rates for Residential Loans (Q491—Q324)

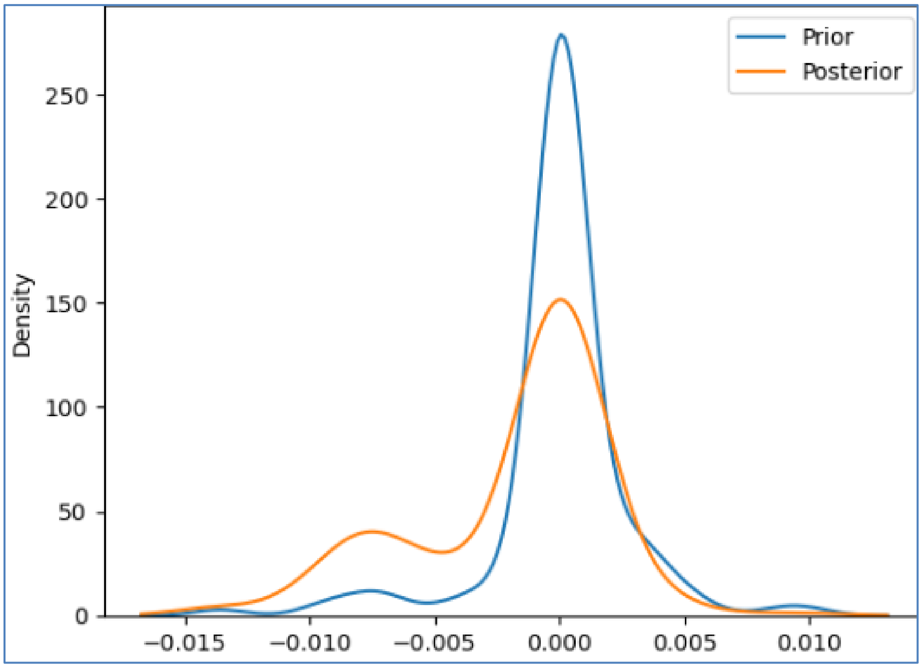
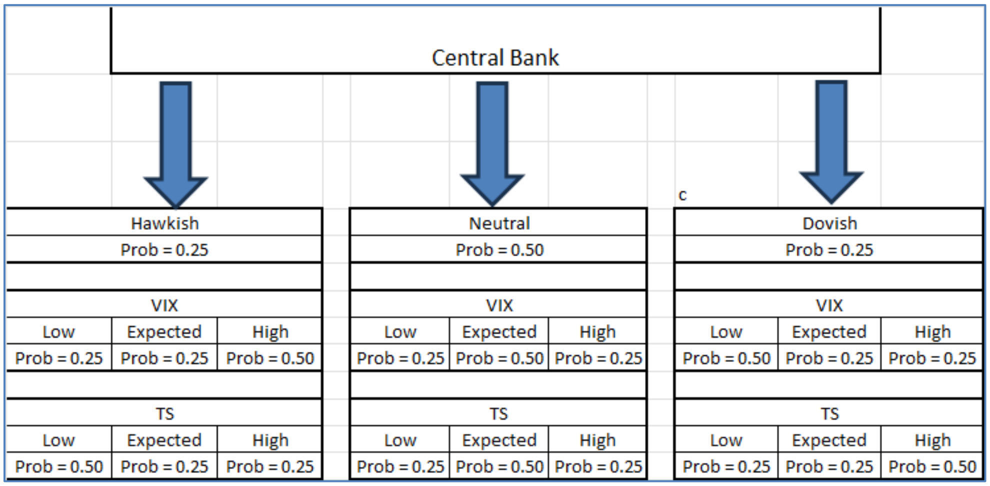


Table 6: Entropy Pooling Model Portfolio Return Distribution Conditional Value-at-Risk Measures - Posterior Risk Factor Shocked Equal and Minimum Risk Weighted, as well as Negative Annual Changes in Y9 Charge-off Rates Shocked Minimum Risk Weighted (FED CCAR 2025 Severely Adverse Macroeconomic Variables (Q125—Q427))

		EP Posterior Equal Weighted	EP Posterior Minimum Risk	Asset Return Views Posterior Minimum Risk
	Conditional VaR	6.84%	6.38%	7.18%
Asset Class Weights	Commercial & Industrial	25.00%	35.13%	30.81%
	Commercial Real Estate	25.00%	32.10%	28.33%
	Credit Card	25.00%	12.19%	18.56%
	Residential Mortgage	25.00%	20.58%	22.30%

Figure 20: Simple Bayesian Network of Central Bank Policy and Macroeconomic Variable Impact Probability Distributions



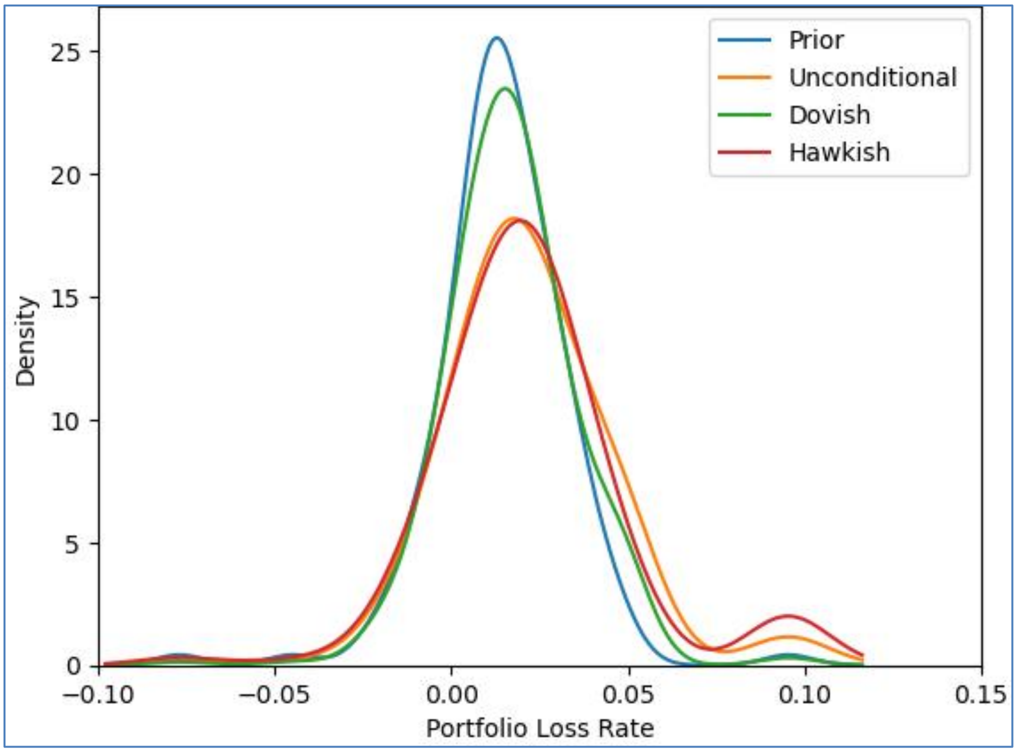
taining a tilt in the direction of the higher Sharpe ratio asset classes that is accentuated for the cases where the views are less severe. This illustrates the value of imposing views on asset class returns that independent of those that pertain to the risk factors, as this lessens the reliance that we put on historical correlations, which prudently brings us back to a more agnostic stance – i.e., minimizing risk by moving away (toward) from asst classes with greater (less) sensitivity to the risk factors in the scenarios is tempered by the extent to which they move independently of the risk drivers due to idiosyncratic factors not in the risk model. Furthermore, the advantage of expressing these asset class return views within the EP framework rather than on an ad hoc basis is that we are forced to be explicit in our risk and return assumptions within the quantitative modeling framework, as opposed to applying add-ons to model results. We note that banks will often base their qualitative add-ons by shocking model output based upon risk factors like PD ratings, which is a way of incorporating information not in the historical risk factors that is similar to this approach, but that depends upon the risk rating framework admitting rating overlays, and may not be as useful for retail asset classes that manage their book based upon credit scores that may be purely quantitative.

Finally, we incorporate the effects of Fed monetary policy into our framework with the application of a simple BN where each node has three states, with the central bank being “dovish”, “expected”, and “hawkish”. The two proxies for policy variables are the TS and VIX, where in a hawkish state the TS flattens and the VIX rises (and the opposite in a dovish state), corresponding to how tight monetary policy flattens the yield curve and leads to greater equity market volatility. We translate the discrete states of the leaf nodes into EP views layered upon the CCAR ST/SA, where we shock the prior mean and standard deviations of the TS and VIX by their historical standard deviations. The state probabilities are specified so that they produce marginal probabilities that correspond to 25%, 50%, and 25% for all variables as shown in Figure 20. With the posterior probability vectors at hand, we can assess the impact of the BN view by comparing the unconditional distribution to the prior and furthermore we can condition on the central bank being either dovish or hawkish and see how this affects the portfolio.

Table 7: Bayesian Network and Entropy Pooling Model - Negative Annual Changes in Y9 Charge-off Rates Shocked Equal Weighted Portfolio Return Prior and Central Bank Scenario Posterior Risk Measures (FED CCAR 2025 Severely Adverse Macroeconomic Variables (Q125—Q427))

Volatility	Prior	2.86%
	Base	2.86%
	Dovish	2.83%
	Hawkish	2.89%
Value-at-Risk	Prior	4.10%
	Base	4.16%
	Dovish	3.91%
	Hawkish	4.28%
Conditional Value-at-Risk	Prior	6.84%
	Base	6.94%
	Dovish	6.52%
	Hawkish	7.14%

Figure 21: Bayesian Network and Entropy Pooling Model - Negative Annual Changes in Y9 Charge-off Rates Shocked Equal Weighted Portfolio Return Prior and Central Bank Scenario Posterior Distributions (FED CCAR 2025 Severely Adverse Macroeconomic Variables (Q125—Q427))



In Table 7 we show the risk measures of the proxy returns, where the EP risk factor shocked equal weighted portfolio is the prior and the posterior is the result of applying the BN central bank scenarios on top of the CCAR ST/SAs, with the corresponding distributions are shown in Figure 21. While the effect of the BN view is minor, the conditional distributions have the expected effect. It is worth noting that the conditional distributions not only assume that the central bank is dovish or hawkish, but also that the BN structure is true, hence as we have done the partial effect of the conditioning should be assessed against the unconditional distribution, not the prior. As a consequence, it is meaningful to use the unconditional distribution as EP prior for computing the conditional distributions. In this example, the difference will be practically irrelevant because the prior and unconditional distributions are very similar.

Conclusion and Future Directions

In this study we have proposed an application to the CCAR ST/SA exercise of a unified methodology that incorporates non-linear scenarios from alternative perspectives in a general non-normal setting for economic and market factors, which is novel in its application to credit risk and a supervisory application of ST/SA, a flexible framework for causal and predictive market scenarios that combines BN and entropy pooling EP, with BN generating a finite set of joint causal views for the relevant risk factors, while EP is used to project each of these stress scenarios over stochastic simulations. The joint view probabilities from BNs have been used as weights for the associated EP probability vectors to compute a single posterior probability distribution. The framework allowed us to implement economic scenarios and perform stress tests conditional on realizations of relevant risk in a purely causal and predictive manner, providing a tool for portfolio, risk and supervisory practitioners managing credit risk and profitability. We tested these methodologies empirically with aggregate banking charge-off data for several lending segments and benchmarked the results against a challenger vector-autoregressive model, finding that the BN and EP approaches produce scenarios that have superior characteristics. Namely, the proposed approach produced more conservative and more accurately estimated portfolio risk measures for the same severely adverse scenarios, with a more parsimonious model that has better fit to the data.

This study has also highlighted the socioeconomic implications of this research. It has been noted that while technologies developed by financial systems can contribute to economic development by providing institutions with useful tools for risk management, when they fail to manage such risks through negligence or faulty methodologies, they can create severe financial crises with devastating social and economic effects. It has been argued that a prime example is the financial crisis of 2008-2009 that transformed the lives of many individuals and families for the worse by throwing them into poverty and exclusion. We have pointed out that the adverse social reverberations of this event were catalysts in many undesirable phenomena (e.g., the rise of populist political ideologies, erosion of trust in societal institutions and the fragmentation of the post World War II geopolitical order.) We have contented that if financial institutions had techniques in their toolkit such as EP and BN, even if applied to the pre-financial crisis risk measurement regime, that the severity of the financial crisis and ensuing fallout would have been greatly ameliorated.

There are several directions in which this line of research could be extended, including but not limited to the following:

- Loan level data;
- alternative asset classes; and,
- international geographies.

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