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Topological Origin of Electric Charge Quantization in Curved Spacetime

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Abstract

The quantization of electric charge is traditionally derived from the consistency condition imposed by magnetic monopoles, as originally demonstrated by Dirac [1]. Subsequent developments in the geometric formulation of gauge theory clarified that magnetic flux quantization follows from the integrality of the first Chern class of principal bundles [2–4].

In this work, we present a complementary derivation based on the global structure of the quantum configuration space. Formulating abelian gauge theory in four-dimensional asymptotically flat spacetime as a sum over inequivalent principal bundles classified by $H^2(M,\mathbb{Z})$, we show that invariance of Wilson line operators under large gauge transformations [7] constrains electric charges to lie in the lattice dual to the magnetic flux lattice. Electric charge quantization therefore emerges as a global consistency condition of the quantum theory, independently of the existence of dynamical monopole solutions.

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Introduction

Electric charge quantization is one of the most fundamental structural properties of gauge theories. In classical Maxwell theory, electric charge may take arbitrary continuous values, and perturbative quantum electrodynamics does not impose discreteness at the level of local dynamics.

Dirac famously showed that the existence of a magnetic monopole implies the consistency condition

$$eg = 2\pi n, n \in \mathbb{Z}$$

thereby enforcing charge quantization [1]. Later developments clarified that gauge fields are more properly understood as connections on principal bundles rather than globally defined vector potentials. In this geometric framework, magnetic flux quantization arises from the integrality of the first Chern class $c_1 \in H^2(M, \mathbb{Z})$[2–4].

In non-abelian gauge theories, topological sectors are classified by homotopy groups, and smooth monopole solutions arise in spontaneously broken theories [5,6]. However, the essential topological structure underlying flux quantization exists independently of the dynamical realization of monopole solutions.

The purpose of this work is to derive electric charge quantization directly from the global structure of the quantum configuration space, defined as a sum over inequivalent principal bundles compatible with the asymptotic structure of spacetime. The derivation relies on invariance under large gauge transformations, as discussed in [7], nd does not assume the existence of singular or smooth monopole configurations.

2. Quantum Configuration Space and Topological Sectors

Let M be a globally hyperbolic, asymptotically flat four-dimensional spacetime.

We consider abelian gauge theory with compact gauge group $U(1)$. Gauge fields are connections on principal $U(1)$-bundles $P \to M$. Such bundles are classified topologically by their first Chern class

$c_1(P) \in H^2(M, \mathbb{Z})$

as discussed in [3,4].

Chern–Weil theory implies that for any closed oriented two-surface $\Sigma \subset M$,

$$\frac{1}{2\pi} \int_\Sigma F \in \mathbb{Z}$$

where $F$is the curvature two-form [3].

A complete definition of the quantum theory therefore requires summing over all isomorphism classes of principal bundles:

$$Z = \sum_{[P] \in H^2(M,\mathbb{Z})} \int_{\mathcal{A}(P)/\mathcal{G}(P)} \mathcal{D}A \;\; e^{iS[A]}$$

The necessity of summing over disconnected topological sectors is familiar from non-abelian gauge theory and instanton physics (see, e.g., [7]).

The magnetic flux lattice is therefore identified with

$$\Lambda_{\text{mag}} \simeq H^2(M, \mathbb{Z})$$

This quantization arises purely from global bundle consistency [2–4].

3. Wilson Lines, Large Gauge Transformations, and Charge Lattices

Electric charge is defined via Wilson line operators:

$$W_q(\gamma) = \exp\left(iq \oint_\gamma A\right)$$

e a

Gauge invarianc requires invariance under both small and large g uge transformations. The structure and physical significance of large gauge transformations are discussed in detail in [7].

In nontrivial bundle sectors, the gauge potential must be defined patchwise, with transition functions satisfying cocycle conditions [2]. Under a large gauge transformation corresponding to a nontrivial element of $H^1(M, U(1))$, the Wilson loop acquires a phase proportional to the magnetic flux through a spanning surface.

Using flux quantization [3,4],

$$\frac{1}{2\pi}\int_\Sigma F = n \in \mathbb{Z}$$

invariance of the Wilson line requires

$$\exp\,(i2\pi qn) = 1$$

which implies

$$qn \in \mathbb{Z}$$

for all admissible flux integers n.

Thus admissible electric charges lie in the lattice dual to the magnetic flux lattice:

$$\Lambda_{\mathrm{elec}} = \mathrm{Hom}(\Lambda_{\mathrm{mag}}, \mathbb{Z})$$

This reproduces the Dirac quantization condition [1], but now derived from global consistency of the full quantum configuration space rather than the assumption of a monopole background.

4. Extension to Non-Abelian Gauge Groups

For compact non-abelian gauge group G, spontaneous symmetry breaking $G \to H$leads to a vacuum manifold G/H. Topological sectors are classified by

$$\pi_2(G/H)$$

which underlies the existence of smooth monopole solutions [5,6].

Magnetic charges span the coroot lattice of G, while electric charges correspond to weights of representations of H. The general structure of charge quantization in non-abelian gauge theories has been studied extensively following 't Hooft and Polyakov [5,6].

Consistency of th global gauge structure requires that electric and m gnetic charges obey lattice duality:

$$\Lambda_{\mathrm{elec}} = \mathrm{Hom}(\Lambda_{\mathrm{mag}}, \mathbb{Z})$$

This duality between weight and coroot lattices is a purely group-theoretic consequence of the global structure of compact Lie groups.

4.1 SU(2) Gauge Theory and Symmetry Breaking

Consider a gauge theory with compact structure group $SU(2)$. The gauge field is a connection
$A_\mu = A_\mu^a T^a$,

with curvature

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu].$$

Suppose the theory contains an adjoint scalar field $\phi^a$with potential $V(\phi)$such that spontaneous symmetry breaking occurs:

$$SU(2) \longrightarrow U(1).$$

The vacuum manifold is then

$$\mathcal{M} = SU(2)/U(1) \cong S^2.$$

4.2 Topological Classification

Topological sectors are classified by the second homotopy group of the vacuum manifold:

$$\pi_2(SU(2)/U(1)) = \pi_2(S^2) = \mathbb{Z}.$$

This integer labels distinct homotopy classes of mappings

$$S^2_\infty \longrightarrow S^2.$$

This classification underlies the existence of smooth monopole solutions in spontaneously broken gauge theories [5,6], although the present argument does not rely on their dynamical realization.

4.3 Path Integral over Non-Abelian Topological Sectors

The quantum th ory must sum over all topological sectors:

$$Z = \sum_{k\in\mathbb{Z}} \int_{\mathcal{A}_k/\mathcal{G}} \mathcal{D}A\, \mathcal{D}\phi \;\; e^{iS[A,\phi]}.$$

Here $k$labels the homotopy class in $\pi_2(SU(2)/U(1))$, and $\mathcal{A}_k$denotes the configuration space in the k-th sector.

The existence of disconnected components of configuration space is a global property of the gauge-Higgs structure.

4.4 Emergence of Charge Lattice

After symmetry breaking, the residual gauge group is $U(1)$. Electric charges correspond to representations of this unbroken subgroup.

However, compatibility with the global gauge structure inherited from the full $SU(2)$theory restricts the admissible representations. The embedding of $U(1) \subset SU(2)$fixes the normalization of the generator and determines a discrete electric charge lattice.

Schematically,

$$q \in g\mathbb{Z}.$$

Thus, charge quantization in the broken phase follows from:

1. The topology of the vacuum manifold.
2. The structure of disconnected configuration sectors.
3. Invariance under large gauge transformations.

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5. Discussion and Outlook

We have shown that electric charge quantization follows from:

1. Flux quantization via the first Chern class [3,4],
2. The necessity of summing over topological sectors in the functional integral [7],
3. Invariance under large gauge transformations [7].

The argument reproduces the Dirac condition [1] without assuming the dynamical existence of monopole solutions, and extends naturally to non-abelian gauge groups following the framework of [5,6].

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