



Mathematical Modeling of Stationary Heat Transfer Through External Enclosing Structures

Said PS (Khuzhaev PS)

Tajik Technical University named after Academician M.S. Osimi, Tajikistan

Citation: Said PS (2026) *Mathematical Modeling of Stationary Heat Transfer Through External Enclosing Structures*. *J. of Sci Eng Advances*. 2(2) 1-6. WMJ/JSEA-134

Abstract

This paper presents a mathematical modeling of steady-state heat transfer through multilayer building envelope structures. One-dimensional heat flow is considered for given indoor and outdoor air temperatures. The equivalent thermal resistance method is used for calculations, taking into account the thermophysical properties of individual building envelope layers and the heat transfer resistance on the internal and external surfaces.

The thermal resistances of the building envelope layers, the total thermal resistance, the heat flux density, and the temperature distribution across the wall thickness and at the layer boundaries are determined. The greatest temperature difference occurs in the layer with the highest thermal resistance, confirming the effectiveness of thermal insulation materials. The results can be used in the design of energy-efficient building envelopes and in thermal engineering calculations for buildings.

***Corresponding author:** Said PS (Khuzhaev PS), Tajik Technical University named after Academician M.S. Osimi, Tajikistan.

Submitted: 24.03.2026

Accepted: 28.03.2026

Published: 15.04.2026

Keywords: Heat Transfer, Enclosing Structure, Thermal Conductivity, Mathematical Modeling, Heat Flow, Thermal Resistance, Temperature Field, Building Thermal Engineering

Introduction

In the modern world, the main focus of new technologies is the desire to reduce harmful emissions into the atmosphere and curtail the use of non-renewable energy resources. One of the most important factors influencing emissions is the combustion of fuel for heating and electricity generation. In response to this

challenge, energy-saving technologies have emerged that reduce energy costs for the operation of buildings and structures.

Using these technologies, external energy consumption for maintaining a comfortable indoor microclimate is minimized, and with passive house building

technologies, it approaches zero. To determine the minimum heating and air conditioning costs for a building, it is necessary to accurately determine the thermal resistance of the building envelope, as well as the temperature distribution within it.

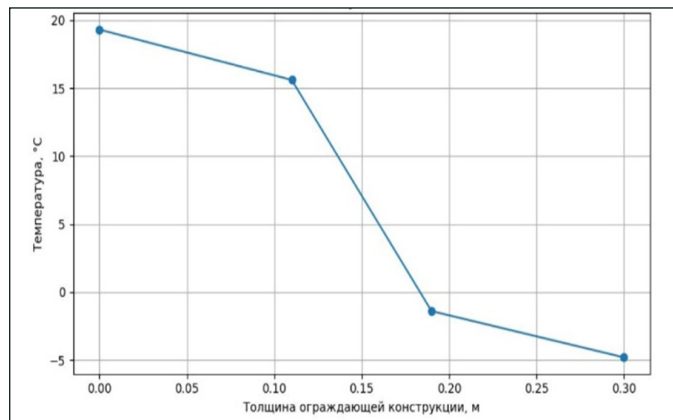


Figure 1: Heat Transfer graph Through the External Enclosing Structure (Dushanbe)

The total thermal resistance of external enclosing structures is one of the key indicators of their thermal insulation properties. It characterizes the structure's ability to prevent the transfer of thermal energy from the building's interior to the external environment and back under steady-state heat transfer conditions. Total thermal resistance is defined as the sum of the thermal resistances of the individual structural layers, as well as the thermal resistances of the internal and external surfaces. The higher this value, the lower the building's heat loss and the higher its energy efficiency.

In the city of Dushanbe, characterized by a sharply continental climate with hot summers and relatively cold winters, thermal insulation requirements for external enclosing structures are particularly important. In winter, it is necessary to ensure minimal heat loss to maintain a comfortable temperature inside the building, and in summer, to reduce external heat inflow, which reduces the load on air conditioning systems [1,2].

The total thermal resistance is calculated based on the climatic parameters of the construction area, such as the estimated outdoor temperature, the duration of the heating season, and the indoor temperature regime. The resulting value is compared with regulatory requirements, allowing for an assessment of the building envelope's compliance with energy

conservation and thermal comfort requirements. (Figure 2)

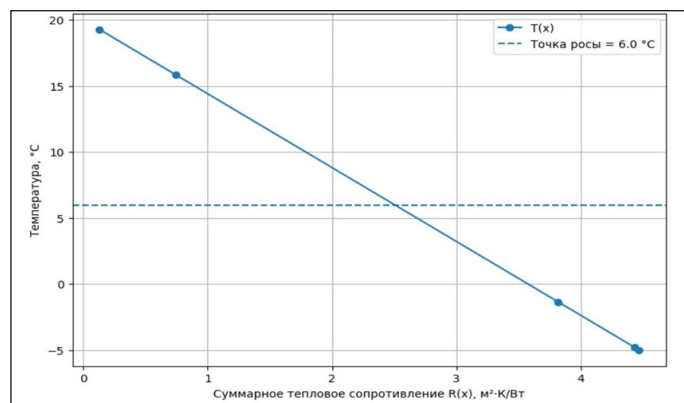


Figure 2: Graph of total Thermal Resistance and dew Point. External Enclosing Structure (Dushanbe)

Modern building energy efficiency requirements necessitate a thorough analysis of heat loss through the building envelope. Exterior walls, roofs, and floors are the primary pathways for heat transfer between indoor and outdoor spaces, and their proper design can significantly reduce energy consumption for heating and air conditioning.

Mathematical heat transfer modeling is widely used in building thermal engineering, allowing for the quantitative assessment of heat flow and temperature field in building envelopes. One of the most intuitive and convenient calculation methods is **the equivalent thermal resistance method**, which takes into account the sequential arrangement of wall layers and the heat transfer resistance of the surfaces.

Physical Statement of the Problem

This paper examines the process of steady-state one-dimensional heat transfer through a flat, multilayer building envelope. Heat transfer occurs from the interior air to the outside environment through successive layers of the structure.

When constructing the mathematical model, the following assumptions were made:

- The thermal regime is considered to be established, the temperature field does not change over time;
- The heat flow is directed perpendicular to the plane of the layers of the enclosing structure, which allows us to consider the problem in a one-dimensional formulation;
- Heat exchange by convection and radiation

- on internal and external surfaces is taken into account using heat transfer coefficients;
- Thermophysical properties of materials (thermal conductivity, density and heat capacity coefficients) are assumed to be constant;
- There are no internal heat sources in the structure.
- The adopted assumptions make it possible to simplify the problem and apply classical methods of building thermal engineering to analyze heat transfer through the enclosing structure [3,4].

Initial Data

For mathematical modeling of heat transfer through the external enclosing structure of a building, the following initial data are specified:

- Indoor air temperature: $t_{int} = 18\text{ }^{\circ}\text{C}$
- Outdoor air temperature: $t_{outside} = -14.2\text{ }^{\circ}\text{C}$
- Temperature difference: $\Delta T = t_{int} - t_{out} = 32.2\text{ }^{\circ}\text{C}$
- The layers of the structure and their thermophysical characteristics (thickness and thermal conductivity coefficient) are accepted according to the assignment;
- Type of enclosing structure - flat multi-layer external wall;
- Heat transfer coefficients on the internal and external surfaces of the structure are accepted as standard;
- Thermal regime - stationary;
- There are no internal heat sources in the structure.

Procedure for Performing Work

- To study the physical formulation of the problem of heat transfer through a multilayer enclosing structure and accept the basic assumptions of the mathematical model.
- Set the internal and external air temperatures, as well as the thermophysical characteristics of the wall layer materials.
- Calculate the thermal resistance of individual layers of the enclosing structure and the heat transfer resistance from the inside and outside.
- Determine the total thermal resistance of the enclosing structure.
- Calculate the heat flux density through a structure at a given temperature difference.
- Determine the temperatures at the boundaries of the enclosing structure layers.

- Analyze the obtained results and draw conclusions about the nature of heat transfer and the influence of the thermal insulation layer on reducing heat loss.

Mathematical Model

Differential Equation of Heat Conduction

The process of heat transfer through the building's external enclosing structure is considered in a stationary one-dimensional setting. In this case, the temperature distribution across the structure's thickness is described by the Fourier differential heat conduction equation.

For the i -th layer of a multilayer enclosing structure, the heat conductivity equation has the form:

$$\frac{d}{dx} \left(\lambda_i \frac{dT}{dx} \right) = 0$$

Where

$T = T(x)$ — temperature, $^{\circ}\text{C}$;

x — coordinate along the thickness of the structure, m;

λ_i — thermal conductivity coefficient of the i -th layer, $\text{W}/(\text{m } ^{\circ}\text{C})$.

Since the thermal conductivity coefficient within each layer is considered constant, the heat conductivity equation is simplified and takes the form:

$$\frac{d^2 T}{dx^2}$$

The solution to this equation is a linear dependence of temperature on the coordinate, which indicates the linear nature of the temperature distribution within each homogeneous layer of the enclosing structure. [3-5].

Boundary Conditions

To correctly solve the differential heat conduction equation, boundary conditions must be specified on the internal and external surfaces of the enclosing structure. In steady-state heat transfer, these conditions take into account convective thermal resistance and the heat flow between the air and the wall surface.

On the Inner Surface of the Structure ($x = 0$):

$$-\lambda \left. \frac{dT}{dx} \right|_{x=0} = \alpha_{BH} \cdot (T_{BH} - T_o)$$

Where:

λ is the thermal conductivity coefficient of the layer material, $\text{W}/(\text{m } ^{\circ}\text{C})$;

α_{BH} — heat transfer coefficient on the inner surface, W/(m² °C);
 T_{BH} — indoor air temperature, °C;
 T_c — wall surface temperature from the room side, °C.

On the outer Surface of the Structure ($x = \delta$):

$$-\lambda \frac{dT}{dx} \Big|_{x=\delta} = \alpha_{Hap} \cdot (T_\delta - T_{Hap})$$

Where:

α_{Hap} — heat transfer coefficient on the outer surface, W/(m² °C);
 T_δ — temperature of the outer surface of the wall, °C;
 T_{Hap} — outside air temperature, °C;
 δ — thickness of the structure, m.

These boundary conditions allow us to take into account the influence of internal and external heat exchange and provide the correct solution for temperature distribution across the thickness of a multilayer enclosing structure [1,6,7].

Thermal Resistance Method

The Equivalent Thermal Resistance Method is widely used for practical modeling of heat transfer through a multilayer enclosing structure. This method allows the multilayer structure to be replaced by a sequence of thermal resistances of layers and surfaces, significantly simplifying the calculation and eliminating the need to solve a differential equation for each layer.

Basic Principles of the Thermal Resistance Layer Method

The thermal resistance of a layer R_i indicates how much a given material impedes heat flow. For a single-layer section, it is determined by the formula:

$$R_i = \frac{\delta_i}{\lambda_i}$$

Where

R_i — thermal resistance of the i-th layer, m² °C/W;
 δ_i — thickness of the i-th layer, m;
 λ_i is the thermal conductivity coefficient of the layer material, W/(m °C).

The higher the value R_i , the greater the layer's ability to retain heat and the less heat loss through that section of the structure. This indicator is especially important when designing multi-layer enclosing structures, as the sequential arrangement of layers with different thermal conductivities determines the

overall thermal efficiency of the wall [6,8].

Heat Transfer Resistance on the internal and external surfaces of the structure:

$$R_{BH} = \frac{1}{\alpha_{BH}}, \quad R_{Hap} = \frac{1}{\alpha_{Hap}}$$

Where

α_{BH} and α_{Hap} — heat transfer coefficients, W/(m² °C).

Total Thermal Resistance of the Structure

The total thermal resistance of an enclosing structure is a key indicator of its thermophysical properties and allows one to assess the structure's ability to impede heat transfer from the interior air to the outside environment. For multilayer walls, the total heat flux is determined not only by the properties of individual materials but also by the resistance to heat transfer on the interior and exterior surfaces of the structure.

Thermal resistance indicates how much a given layer impedes heat flow. The higher the R_i value, the more effectively the layer retains heat. This parameter is especially important when designing multi-layer enclosing structures, where the sequential arrangement of materials with different thermal properties determines the overall thermal efficiency of the wall.

For practical calculations, the total resistance of a structure takes into account not only the resistance of individual layers, but also the thermal resistance of the surface due to convection and radiation:

$$R_\Sigma = R_{BH} + \sum_{i=1}^n R_i + R_{Hap}$$

Heat flow Density through the Structure

$$q = \frac{T_{BH} - T_{Hap}}{R_\Sigma},$$

Where

T_{BH}, T_{Hap} — indoor and outdoor air temperatures, respectively.

Temperatures at the Layer Boundaries

For a multilayer enclosing structure, it's important to determine the temperatures at the boundaries of each layer, as this allows for heat distribution analysis and identification of layers with the greatest temperature differences. The temperature at the boundary of the kth layer is calculated using the formula:

$$T_k = T_{BH} - q \sum_{i=1}^k R_i, k = 1, 2, \dots, n$$

Where

T_k — temperature at the boundary of the k-th layer.

T_{BH} — indoor air temperature, °C;

α — heat flow density through the structure, W/m²;

R_i — thermal resistance of the i-th layer, m² °C/W.

This formula allows one to determine the temperature profile across the entire thickness of the structure. The greatest temperature differences are typically observed in the layers with the highest thermal resistance, which most often corresponds to the insulation. Analyzing the temperature at the layer boundaries is necessary to assess the effectiveness of the thermal insulation and prevent possible condensation within the structure [9].

The thermal resistance method allows for a clear determination of temperature distribution throughout the entire thickness of a structure and the identification of layers with the greatest temperature differences. This approach is widely used in building thermal engineering for calculating multi-layer walls, roofs, and other enclosing structures [8].

The method allows you to clearly determine the temperature distribution across the wall thickness and identify the layers with the greatest temperature difference (usually insulation).

Simulation Results

As a result of mathematical modeling of heat transfer through a multilayer external enclosing structure, the following main results were obtained:

The Heat Flow Density through the structure was found to be constant throughout the wall's thickness, consistent with the laws of steady-state heat transfer. This confirms the validity of the chosen one-dimensional model and the thermal resistance method.

Temperature Distribution Across Layers showed that the greatest temperature difference is observed in the layer with the highest thermal resistance—usually the insulation. This result confirms the effectiveness of thermal insulation materials in reducing heat loss through the structure.

The average Temperature of the Structure was approximately 3.6°C, which corresponds to the theoretical value for a linear temperature distribution

between the indoor and outdoor air. This indicator allows one to assess the thermal state of the structure and serves as the basis for further thermal engineering calculations, including determining the dew point and testing for moisture condensation [4,8,10].

Thus, the conducted modeling allows us to clearly assess the impact of different wall layers on heat loss and use the results for designing energy-efficient enclosing structures [11-20].

Conclusion

Mathematical modeling of heat transfer through a multilayer enclosing structure demonstrated that the developed model can effectively solve practical engineering problems. It can be used to:

- determine the heat flow through the structure;
- obtain temperature distribution across the wall thickness;
- evaluate the effectiveness of the thermal insulation layer;
- use the results for designing enclosing structures and conducting thermal calculations for buildings.

The thermal resistance method used in this study is a clear and convenient tool for practical heat transfer analysis. It allows for quick assessment of temperature distribution and heat flow differences in various structural layers and complies with modern building thermal engineering regulations.

Thus, the proposed approach can be used in the design and optimization of enclosing structures to improve the energy efficiency of buildings.

References

1. Khozhaev PS, Ismatullozoda J I, Hasanov F N (Year) Energy-efficient wall enclosures of buildings with high thermal insulation characteristics. Scientific Journal Water Resources, Energy and Ecology 3: 109-118.
2. Khozhaev PS (2022) Assessment of thermal insulation properties of external enclosures of rural residential buildings. Proceedings of the International Scientific and Practical Conference "Integration of Science, Education and Enterprises in the Production of Modern Building Materials and Products 203-206.
3. Pochchoev M M, Khuzhaev PS (2019) Analysis of thermal insulation properties of external

- enclosures of rural residential buildings. Polytechnic Bulletin (Series: Engineering Research) 4: 140-144.
4. Kholmurodov T R, Pochoev M M, Jabborov N M (2021) Efficient use of solar heating systems in the Republic of Tajikistan. Proceedings of the Regional Online Scientific and Practical Conference “Modern Science: Current Issues, Achievements and Innovations” 70-76.
 5. Suleimanov A A, Khuzhaev, P S, Suleimanov Z A, Pochchoev M M (2015) Reducing the energy consumption of a building by using thermal insulation materials. Bulletin of the Tajik Technical University 2: 122-126.
 6. Fokhakov A S, Kobuliyev Z V, Khuzhaev P S, Saidaliyev A A (2019) Transport and technological service system and its impact on the environment. Bulletin of Science and Practice 5: 193-202.
 7. Kholmuratov T R, Saidova M S (2024) Combined heating system of a solar house and its mathematical modeling. Proceedings of the International Scientific and Practical Conference “New Directions of Development of Science in Technical Branches” 258-266.
 8. Law Tajikistan in about energy (AMO JT , 2000 , No. 11, item .504 ; 2005 , No. 12. item .633; 2007 , No. 6, item .436; 2007 , No. 7, item .672; 2009 , No. 9-10, item .549; 2011 , No. 6, item .440, 2013 , No. 7, item .526, No. 12, item .898)
 9. Khujaev P S (2023) Factors affecting the decrease and improvement of thermal and physical characteristics of building envelopes. Science and Innovation Science Section: Geology and Technical 1: 167-176.
 10. Khuzhaev P S, Pochchoev M M (2021) Heat and humidity conditions of external enclosures using the example of an external wall with mineral wool block insulation. Materials of the International Scientific and Practical Conference Modern Problems of the Metallurgical Industry 283-290.
 11. Law Friday (2013) Tajikistan: On Measures for Energy Saving and Efficiency. Journal of Tajikistan 651: 8-9.
 12. Khujaev JH, Saidzoda JH, Ganizoda J Sh, Safarzoda DG (2024) Designing energy-efficient buildings using natural wall materials. Message Polytechnic Department: Research Engineering 1: 219-225.
 13. Khuchaev P S, Ismatullozoda Ch I, Sulaymonova NA (2025) Ecological ways to increase the energy efficiency of buildings. Science and Innovation Department of Geological and Technical Sciences 1: 213-220.
 14. Khujaev P S (2023) Factors affecting the decrease and improvement of thermal and physical characteristics of building envelopes. Science and Innovation Science Section: Geology and Technical 1: 167-176.
 15. Khujaev PS, Boboev SM, Saidgufonov NP, Saidgufonov BP (2023) Testing of the structural design of a residential building with diverse materials: thermal and physical characteristics. Ilm va Innovatsia Section: Geology and Technical Sciences 1: 195-200.
 16. Khuzhaev PS, Shokirov RM, Ismatullozoda D I, Nurov M T (2023) Improving the energy efficiency of external enclosing structures in hot climates. Science and Innovation (Series: Geological and Technical Sciences) 4: 68-75.
 17. Khuzhaev P S, Ismatullozoda J I, Abdughaniyev A M, Sitamov M S (2024) Study of the characteristics of modern thermal insulation materials. Science and Innovation (Department of Science: Geology and Technical) 1: 90-98.
 18. Tabunshchikov Yu A, Naumov A L (2012). Energy efficiency in construction: harmonization of the domestic regulatory framework. AVOK 6.
 19. Livchak V I (2012) Another argument in favor of increasing the thermal protection of buildings. Energy Saving 6.
 20. Livchak V I, Zabegin AD (2011) Bridging the gap between energy saving policy and real energy resource savings. Energy Saving 4.