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# *The Riemann Hypothesis*

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## *Abstract*

*There are seven Mathematical unsolved problems, which are opened and introduced by the Clay Mathematics Institute in 2000. These problems are the Birch and Swinnerton-Dyer conjecture, Hodge conjecture, Navier-Stokes existence and smoothness, P versus NP problem, Riemann hypothesis, Yang-Mills existence mass gap, which are opened yet. It has been officially given the name Millenium Problem, which consists of seven unsolved mathematical problems. Hence Poincare conjecture has carried at the Millennium Meeting on May 24 2000. The Riemann Hypothesis, is one of them, which is well-known complex mathematical problems. I studied one of them and introduced it in this paper. I investigated Riemann hypothesis, which is important concept of mathematics. In addition, it is Hilbert's eight problem too, which is still considered an important unsolved problem a century later. This problem has been well-known unsolved mathematical problem after it was investigated by Bernhard Riemann since 1860. I think that every problem can be solved and it is important to point out that it is possible to consider at unsolved mathematical problems in ordinary mathematical plane in order everyone can understand this problem and there is not any unsolved problem. We have to increase the motivation for adding a strategy and mathematical skills. We meet unexpected problems every day. Sometimes we have to use such mathematical methods. Here I introduced new review for the Riemann Hypothesis. First, I studied infinite series, then I investigated the non-trivial zeros of the complex number. In addition, I gave relationship between infinite series and algebraic curves.*

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## Introduction

One knows, that there are several unsolved mathematical problems. One of them the Riemann Hypothesis, which belongs to Millennium Problems and it is also Hilbert's eighth problem. First of all, I have to note Swiss mathematician, physicist, astronomer, logician, geographer, and engineer Leonhard Euler, who was born in Basel. Leonhard Euler has studied graph theory and introduced some methods for other branches of mathematics, such as analytic number theory, complex analysis and infinitesimal calculus. Much of modern terminology and mathematical notations have founded by him.

In addition, Georg Friedrich Bernhard Riemann was a German mathematician who has investigated scientific works to analysis, number theory and differential geometry. He worked at several branches of real analysis, he has given first exact formulation of the integral, which called the Riemann integral and he studied Fourier series too. Riemann surfaces is one of his contributions to complex analysis [1-3].

## Materials and Methods

The notion of infinite series have been investigated in the 17th century. It was difficult to understand which series were convergent or divergent by Mathematicians in the 18th century. These mixed reviews led to various solutions. A.

L. Cauchy introduced an exact definition of the notion of convergence of series in 1821. After it, a lot of mathematicians have mostly worked at the concept of the convergent series. However, the concept of divergent series appeared in many problems in analysis, the study of such series could develop, and following the definition of their sum was given. I have to note that, some results were given by L. Euler, N. Abel, and others, it was during on the 19th century. Then new methods of summation of divergent series were studied systematically. This study created following new branch of mathematics. Then some important methods of summation of divergent series are studied. Hence general foundation of Cesaro was developed of the theory of linear transformations.

In addition, infinite series, the sum of infinitely many numbers related in a given way and listed in a given order. Infinite series are useful in mathematics and in such sciences as physics, chemistry, biology, and engineering [4-14].

## Results

A complex number z0 is a zero for the holomorphic function f if f(z0) = 0. In particular, analytic continuation shows that the zeros of a non-trivial holomorphic function are isolated. In other words, if f is holomorphic in Ω and f(z0) = 0 for some z0 ∈ Ω, then there exists an open neighborhood U of z0 such that f(z)≠ 0 for all z ∈ U − {z0}, unless f is identically zero. We start with a local description of a holomorphic function near a zero. Indeed, the zeros of a function ζ s are values of the variable s such that the values satisfy the equation ζ s = 0. Let us understand the meaning of the zeros of a function given below. If given function ζ s is equal to zero, then it means that this series is convergent.

$$\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{n^s}=\frac{1}{1^s}+\frac{1}{2^s}+\frac{1}{3^s}+\cdots$$

$$=\frac{1}{1^{a+bi}}+\frac{1}{2^{a+bi}}+\frac{1}{3^{a+bi}}+\cdots =$$

$$\frac{1}{1}+\frac{1}{2^a\times 2^{bi}}+\frac{1}{3^a\times 3^{bi}}+\cdots =$$

$$1+\frac{1}{2^a\times 2^{bi}}+\frac{1}{3^a\times 3^{bi}}+\cdots =$$

$$\sigma_{n=1}^{\infty}\frac{1}{n^a}\cdot\frac{1}{n^{bi}}.$$

According to the p-series test, a series $\sigma_{n=1}^{\infty}\frac{1}{n^p}$ converges if p>1, and if $p=\frac{1}{2}$ then this series diverges. Using the integral test, we get

$$\int_1^{\infty} f(x)dx=\lim_{t\to\infty}\int_1^{t} f(x)dx=$$

$$\int_1^{\infty}\frac{1}{n^x}dx=\lim_{t\to\infty}\int_1^{t}\frac{1}{n^x}dx=$$

$$\lim_{t\to\infty}\left(-\frac{1}{n^x}\times\frac{1}{lna}\Big|_1^t\right)=\lim_{t\to\infty}\left(-\frac{1}{lna}\times\left(\frac{1}{n^t}-\frac{1}{n}\right)\right) =$$

$$-\frac{1}{lna}\times\lim_{t\to\infty}\left(\frac{1}{n^t}-\frac{1}{n}\right)=-\frac{1}{lna}\left(\frac{1-n^{t-1}}{n^t}\right)=0$$

where n is any number. It means that this series is convergent. In addition, it means that given function ζ s has zeros. But given function was originally defined as the infinite series, which each term is not equal to zero. If every term tends to zero, then the sum will tend to zero [5-9]. Following

$$\zeta(s) = \sigma_{n=1}^{\infty}\frac{1}{n^s} \Rightarrow 0: \frac{1}{n^s} \Rightarrow 0$$

That is,

$$\frac{1}{\infty}+\frac{1}{\infty}+\frac{1}{\infty}+\cdots = 0+0+0+\cdots = 0.$$

We see 1 ns is an exponential function, where n is any prime number. Ifs→∞then the function 1 ns →0. There for ewe have to find such complex numbers that s→∞.We knew that, the infinity concept in the complex plane is an infinite number, whose complex argument is unknown or undefined. Everyone knows that, the sphere named the Riemann sphere is a sphere, which belonging to extended complex plane, that is the complex plane and one point at infinity. With aid of the Riemann model, the point ∞ is just near to very large numbers. In addition, geometrically the set of extended complex numbers is called the Riemann sphere. It gives by following form

$$x^2+y^2+z^2=z$$

If $x^2+y^2=c$ is a constant then we get $z^2+c=z$. This equation gets the Mandelbrot set, which defined in the complex plane with complex numbers c. Indeed, the function $f_c(z)=z^2+c$ does not diverge to in finity when iterated start in gat z=0, i.e., and here giving sequence $f_c(0)$, $f_c(f_c(0))$, etc., remains bounded in absolute value. We have to note that, for each sample point, the sequence $f_c(0)$, $f_c(f_c(0)),\ldots$ goestoinfinity: $S: x^2+y^2+z^2=z$.

Hence

$$x^2+y^2+z^2=z$$

$$x^2+y^2=z-z^2$$

$$x^2+y^2\geq 0 \rightarrow z-z^2\geq 0$$

$$z^2-z\leq 0 \rightarrow z(z-1)\leq 0$$

$$z\in(0;1)$$

Otherwise

$$z^2-z\leq 0$$

$$(a+bi)^2-(a+bi)\leq 0$$

$$a^2+2abi-b^2-(a+bi)\leq 0$$

$$(a^2-b^2-a)+(2ab-b)i\leq 0$$

$$\begin{cases}(a^2-b^2-a)\leq 0\\ (2ab-b)\leq 0\end{cases}$$

Solving second equation we have following

$$b(2a-1)\leq 0$$

$$\begin{cases}b\geq 0\\ 2a-1\leq 0\end{cases} or \begin{cases}b\leq 0\\ 2a-1\geq 0\end{cases}$$

$$\begin{cases}b\geq 0\\ a\leq\frac{1}{2}\end{cases} or \begin{cases}b\leq 0\\ a\geq\frac{1}{2}\end{cases}$$

In addition, solving second equation we get

$$(a^2-b^2-a)\leq 0$$

(a2−a)≤b2

We get $a=\frac{1}{2}$, $b\in R$.

## Discussion

The purpose of mathematics, for many of today's developing math skill and knowledge of some students. But it is very difficult. İt looks like, speaking unknown foreign language. As well as, they have difficulty translating several words, understanding their meaning and in addition, how to solve these problems. It is not enough to know how they understand given problem must be solved, they must be know why they must be solved these problems. We have to increase the motivation for adding a strategy, mathematical skills and some new methods too. We meet several problems every day. It is not enough to use knowing methods. Here I introduced new method for the Riemann Hypothesis. Taking relationship between infinite series and algebraic curves, one can use this problem with easy way.

## Conclusion

Mathematical analysis is one branch of mathematics, that deals with numbers, functions, spaces, series (such as arithmetic series, geometric series), infinity series, power series and Taylor series. With aid of the discovery of the numerical series and infinity series by Cauchy, Euler, Fourier and Wilhelm Leibnitz and with applications throughout the sciences and in areas such as finance, economics, and sociology Mathematical Analysis has developed and grown. My aim is the study of some unsolved mathematical problem by ordinary mathematics in order to explain given problem in an easy way. First of all, let me underline that the greater part of the men studied this question and a lot of peoples read these articles which were considered infinite series. Let us say thank to Euler for such interesting and important mathematical problems [15-20].

## Discussion

I have to note that, every mathematicians agrees that the Riemann Hypothesis is more difficult, interesting and unsolved problem. It gives relation-ships between various branches of math. In addition, the hypothesis and the zeta function were investigated by German mathematician Bernhard Riemann, he has introduced them in 1859. Studying prime numbers and their properties, he could obtain them. Solving this problem other unsolved problems can unlock an avalanche of their following progress. It will be developed new branches of Mathematical Analysis and Number Theory.

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